MATHEMATICAL LEARNING PROFILES AND DIFFERENTIATED TEACHING STRATEGIES

By Maria R. Marolda and Patricia S. Davidson

Are there particular mathematical profiles that characterize how students learn mathematics?

How does the understanding of mathematical learning profiles translate into better instrutional opportunities in mathematics?

A primary consideration in the teaching of mathematics is the recognition that students bring to the mathematics classroom a wide range of abilities and learning approaches. Extensive instructional and clinical investigations during the past 20 years, as well as a detailed research study (Davidson, 1983), have revealed that students' learning profiles are marked by different constellations of relative strengths and relative weaknesses with which students face the world of mathematics. Indeed, it is this study of *differences*, rather than a definition of explicit deficits, that provides a more useful approach to understanding students' effectiveness or inefficiencies in learning mathematics. Moreover, an understanding of differences is also informative in fashioning instructional approaches that are compatible with the various learning profiles that exist in the mathematics classroom.

Child / World System

Learning profiles in mathematics can best be understood by considering a *Child/World* system (Bernstein & Waber, 1990) that characterizes the reciprocal relationship of the developing child and the mathematical world in which the child must function. The construction of a *Child/World* system focusses on differences among learners as well as differences in the demands of what is to be learned. It then explores instances of *match* and *mismatch*. In the consideration of differences among students, the critical question becomes, "When does a learning difference render a student learning disabled?" A learning "disability" in mathematics may be thought of as the occurrence of multiple "mismatches" and the inability to overcome those mismatches. A tantalizing issue then becomes whether specific approaches or strategies could be used so that the mismatches are minimized and the disability is resolved or disappears.

It is important to recognize that the diagnostic process in education is quite different from the diagnostic process in medicine. Whereas the medical diagnostician is looking to uncover what is wrong and what the patient can't do, the educator must strive to uncover the student's strengths and what the student can do. The goal of the educator is to find those strengths that can be used to address the weaknesses and difficulties inherent in students' learning profiles.

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In focussing on the *Child* in the *Child/World* system of mathematics, a multidimensional view must be taken and a variety of parameters considered. Specifically, the following factors should be explored in order to understand a student's Mathematical Learning Profile:

- the presence of specific developmental features that are prerequisite to specific mathematics topics;
- the preferred models with which mathematical topics are interpreted;
- the preferred approaches with which mathematical topics are pursued;
- memory skills that affect students' ability to participate in mathematical activities;
- language skills that affect students' ability to participate in the mathematical arena.

Developmental Features of Mathematical Learning Profiles

A definition of a student's mathematical learning profile should incorporate an appreciation of the developmental maturity of students at various ages. There are many developmental milestones in terms of mathematical readiness for dealing with numerical, spatial and logical topics. For numerical concepts, the developmental milestones consist of an appreciation of number, the concept of number (enumeration/cardinality), conservation of number, one to one correspondence and the principles of class inclusion. For spatial concepts, the construct of space, conservation of length and conservation of volume must be considered. For logical thought, developmental milestones include the concepts underlying classification, seriation, associativity, reversibility and inference. Most children between the ages of four and eight have acquired these milestones.

Recent clinical investigation and teaching practice have suggested that the concept of place value might also be developmentally mediated (Marolda & Davidson, 1994). That is, an appreciation of place value depends more on the state/age of the child than on specific teaching experiences. If the child is not cognitively ready to deal with place value, then the concept of place value cannot be formally or meaningfully developed, despite teaching efforts. The formal concept of place value seems to be established for most children between the ages of six and eight. The appreciation of formal place value concepts is of particular importance since they are necessary prerequisites for the understanding of larger quantities and the pursuit of multi-digit computation.

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Preferred Models and Preferred Approaches of Mathematical Learning Profiles

Mathematical situations can be interpreted with concrete, pictorial, or symbolic models. For a particular student, a specific interpretation might be more comfortable and meaningful. Among concrete models, further distinctions can be made. Within the concrete mode, students may prefer set (discrete) models, such as counters, while others appreciate perceptually driven (measurement) models, such as Cuisenaire rods.

The ways in which students process or approach mathematical situations follow two distinct patterns (Marolda & Davidson, 1994). Some students process situations in a linear fashion, building forward to an exact final solution. Sometimes, these students are so focused on the individual elements that the overall thrust or goal is obscured. This style of processing is often characterized as a sequential, step-by-step approach. For other students, a careful building up approach holds little inherent meaning. Such students prefer to establish a general overview of a situation first and then refine that overview successively until an exact solution emerges. Such students may be prone to imprecision and tend to lack appreciation of all relevant details. This style of processing is often described as global or gestalt.

Incorporating these inherent preferences in terms of models and processing has led to the definition of two distinct learning profiles in mathematics, *Mathematics Learning Style I* and *Mathematics Learning Style II* as reviewed in Table 1 (Marolda & Davidson, 1994). Moreover, it is possible to describe mathematical concepts and procedures that are inherently compatible with each learning profile.

To be full and successful participants in mathematics, students must learn to mobilize both Mathematics Learning Style I and Mathematics Learning Style II. The student with special learning needs, however, is often limited to one learning style alone and is unable to mobilize skills and strategies associated with the alternative learning style. For success, teachers must translate activities into the student's operating style, building a scaffold that integrates the areas of strengths and weaknesses so that they complement one another and lead to the acquisition of mathematical concepts and procedures in a meaningful way.

Mathematics Learning Style I	Mathematics Learning Style II
Preferred Models for Number: • Set Models	*Preferred Models for Number:* • Perceptual (Measurement) Models
Preferred Approaches: • Linear, step by step • Often relies on verbal mediation	*Preferred Approaches:* • Deductive, global • Often relies on successive approximations
Topics Approached with Ease: • Counting forward & counting-on • Concepts of addition & multiplication • Pursuit of calculation procedures • Fraction concepts interpreted in verbal terms • Geometric Shapes: Emphasis on naming	*Topics Approached with Ease:* • Counting backward • Concepts of subtraction & division • Estimation • Fraction concepts interpreted in a variety of visual models • Geometric Shapes: Emphasis on spatial relationships and manipulations
Topics of Particular Challenge: • Broader concepts and overarching principles • Estimation strategies • Appreciation of appropriateness of solution generated • Selection of arithmetic operation in word problems; difficulty switching between operations in a set of word problems • Concept of a fraction • More sophisticated geometric topics • Requirement for flexible or alternative approaches	*Topics of Particular Challenge:* • Appreciation of all salient details of multi-step procedures or word problems • Pursuit of multi-step calculation procedures • Relevance of exact solutions; prefers to guess • Follow through to exact solutions in word problems, despite correct choice of operation • Formal fraction operations, despite comfort with underlying fraction concept • Requirement to describe approach in exacting verbal terms • Insistence on a single, specific approach

Table 1

The following charts (Tables 2 & 3; as shown on pages 13 and 14) offer more explicit features of each of the Mathematical Learning Styles and can be helpful in recognizing them and teaching to them.

Memory Skills as a Feature of Mathematical Learning Profiles

Often students are characterized as having difficulty in mathematics because they "can't remember." The attribution of mathematical difficulties to a global memory deficit is somewhat simplistic. Cognitive psychologists (Holmes, 1988) suggest that memory issues, in general, are very complex.

> In evaluating a child's recall of materials, the clinician should recognize the various components of the process loosely called memory: registration of the stimulus, encoding, organization, storage and retrieval...Learning disabled children, however, are constantly described in the psychological and educational literature as having memory deficits of various types, usually visual or auditory (short term or otherwise). In almost all cases, the impairment involves either the initial encoding or the effective retrieval of information. (p.189)

In mathematics, it is particularly important to consider the distinction between encoding and retrieval aspects of memory. Is the student having difficulty remembering the fact or procedure because it was never properly understood and therefore not encoded for storage in memory? Or is the student having difficulty remembering the fact or procedure because it cannot be accessed from the student's repertoire of learned skills?

Four specific memory skills are important in mathematics:

- retrieval of solutions to one digit facts;
- the recall of the sequence of multi-step procedures;

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- visual memory of perceptual / geometric stimuli;
- recall of mathematical data presented auditonally.

In terms of retrieval difficulties in the production of solutions to one digit facts, mathematically it may be more important to consider if the solution is produced efficiently rather than automatically. The distinction that is important is whether the retrieval is automatic or efficient. Difficulties with the retrieval of one digit facts may be supported by alternative strategies that are compatible with a student's inherent learning style and result in more efficient production of solutions. In the example, 8 + 6, a student with Mathematics Learning Style I would be most efficient turning to counting on strategies: 9,10,11,12, 13...**14**! or strategies that build 10s: 8 + (2+4) = 10 + 4 = **14**! A student with Mathematics Learning Style II would be most efficient turning to related facts, e.g. doubles, 8+8=16, so 8+6=14 ...2 Less! Or 6+6=12, so 8+6=14...2 More!

In dealing with multi-step procedures, the recall of the organization of the specific steps relies on an understanding of the conceptual foundations driving the procedure. By offering alternative approaches that appeal to a specific learning style, the procedure is better understood and more easily pursued. In dealing with the multiplication problem 23 x 14, a student with Mathematics Learning Style I would turn to a successive addition approach or the formal algorithm. Further supports to remembering the steps of procedures include encouraging verbal mediation techniques, developing verbal and visual flow charts that can be used as referents, and developing mnemonics to cue each step. In contrast, the student with Mathematics Learning Style II would turn to the definition of multiplication as an area and would then combine the area of the four subregions to determine the final solution. Further supports would be ...imation techniques made iteratively ...of, once an initial estimate is made, the use of a calculator for an exact solution. With firm understanding established, the procedure is more effectively encoded. That understanding, however, may emerge from different approaches.

In dealing with geometric designs, students need to use visual memory skills. With visual memory difficulties, students may find the building and copying of geometric designs challenging. To support visual memory difficulties students might be encouraged to interpret geometric designs in verbal terms. Difficulties in visual memory can also manifest themselves in non-geometric situations, such as difficulties orienting written digits, difficulties aligning numerals in written procedures, and difficulty organizing a page of problems. Copying problems from the text and the board or interpreting data presented on a computer screen may also be difficult. In response, copying requirements should be minimized, while graphic organizers may be offered to support the copying that is required.

Students with auditory memory difficulties are challenged when required to remember all relevant data presented in instruction, remember the overall outcome sought, remember directions, or remember all the relevant information in word problem situations presented verbally. These students may be supported by offering directions in visual formats as well as by offering written directions and / or allowing students to write down the directions and then referring to the written text as needed. Interestingly students with apparent auditory memory issues are often confused with students whose primary difficulties are in language where memory difficulties are secondary to specific language processing issues.

Language Issues

Language skills, both oral and written, are important in mathematics in terms of:

- word retrieval skills;
- verbal formulation requirements;
- comprehension requirements.

They become an issue when students are required to retrieve the names of coins, geometric shapes or other mathematical terms, when they are asked to explain their solutions or approaches, when they must deal with lengthy verbal presentations typical of classroom instruction, and when they are faced with word problems. These language demands have become more prominent in mathematics as education curricula and textbooks have encouraged teachers to ask students for explanations or justifications of their approaches. Moreover, teachers have been encouraged to ask students to take responsibility for their own learning by reading printed materials or texts. These newer emphases pose particular challenges for students with language difficulties.

In order to address word retrieval difficulties in mathematics, students might focus primarily on the values of the coins rather than their specific names, might be encouraged to draw geometric shapes rather than name them and might be offered recognition formats when dealing with mathematical definitions. Retrieval issues are further supported by minimizing confrontational, fast answer situations.

Students with verbal formulation issues often have difficulty describing their approaches or in portfolio work where approaches must be written down. To support these students, alternate forms of communication should be encouraged, including demonstrations with physical models and use of pictures or diagrams to describe solution processes.

Students with comprehension difficulties often have difficulties with directions and with reading texts or word problems. They often can't get started with classwork, mistakenly suggesting they have attentional difficulties. These students benefit from careful monitoring of new presentations and having word problems read to them. Such students can be supported by teachers presenting content in meaningful "chunks" that are then carefully linked together. In terms of word problems, the situations can be presented verbally rather than requiring reading. Students should then be encouraged to draw pictorial interpretations to represent the situation and data involved.

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Mathematical Learning Profiles and Differentiated Teaching Strategies

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Mathematics Learning Style I		
Cognitive & Behavioral Correlates	**Mathematical Behaviors**	**Teaching Implications & Strategies**
• Highly reliant on verbal skills	• Approaches situations using recipes; "talks through" tasks • Interprets geometric designs verbally	• Emphasize the meaning of each concept or procedure in verbal terms. • Build on subvocalization strategies to direct procedures.
• Tends to focus on individual details or single aspects of a situation • Sees the "trees," but overlooks the "forest"	• Approaches mathematics in a mechanical, routine based fashion • Overwhelmed in situations in which there are multiple considerations, such as in multi-step tasks • Can generate correct solutions, but may not recognize when solutions are inappropriate • Difficulties "checking" work; must re-do entire problem • Difficulties choosing an approach in word problems • Difficulties appreciating larger geometric constructs because of an emphasis on component parts	• Highlight concept /overall goal. • Break down complex tasks into salient units and make linkage between units explicit. • Build simple estimation strategies; encourage two final steps to each calculation problem: "Does this answer the question?" and "Does the solution seem right?" • Encourage students to rewrite or state problems in their own words. • Develop metacognitive strategies to analyze word problem situations. • Encourage parts to wholes approach in building geometric figures and explicit descriptions of the overall design that emerges.
• Prefers HOW to WHY	• Prefers numerical approach over manipulative models • Needs drill and practice to establish procedure before considering applications or broader conceptual meaning	• Link manipulative model on a step-by-step basis to the numerical procedure. • Once procedure is secure, relate math topics to relevant real life situations.
• Relies on a defined sequence of steps to pursue a goal • Reliant on teacher for THE approach • Lack of versatility	• Prefers explicit delineation of each step of a procedure and linkage of steps one to another • Vulnerable when there are multiple approaches to a single topic • Overwhelmed by multiple models or multiple approaches • Prefers linear approaches for arithmetic topics	• Offer flow chart approaches. • Help students create handbooks with procedures described in their own words. • Choose one manipulative model or approach to develop a wide range of topics; avoid switching models or approaches too quickly. • Don't emphasize special cases; rather develop an over-riding rule that applies to all cases; e.g. for the addition and subtraction of fractions with unlike denominators, develop a single process using the product of the denominators in all cases, even if it is not the least common denominator. • Give explanations before or after procedure, but not while student is pursuing procedure. • Use counting on techniques for addition facts and missing addend techniques for subtraction facts. • Interpret multiplication as successive additions.
• Challenged by perceptual demands	• Difficulties with more sophisticated perceptual models, such as Cuisenaire rods • Geometric activities may be challenging, especially in three dimensions. • Difficulties interpreting analog clocks • Difficulties distinguishing coins, especially nickel and quarter • Difficulties organizing written formats	• Emphasize set (discrete) models for counting, such as money or counting chips. • Translate perceptual cues in terms of verbal descriptions.
• Prefers quizzes or unit tests to more comprehensive final exams	• May be able to complete the most difficult example in a set of examples relying on the same concept/skill, but has difficulty switching to a new topic or new approach	• Spiral all topics to keep them current.

Table 2

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Mathematics Learning Style II		
Cognitive & Behavioral Correlates	**Mathematical Behaviors**	**Teaching Implications & Strategies**
• Prefers perceptual stimuli and often reinterprets abstract situations visually or pictorially	• Benefits from manipulatives • Loves geometric topics	• Offer a variety of models; introduce perceptual models, such as *Base Ten Blocks* or *Cuisenaire Rods*, to support calculations. • Emphasize geometry as a vital part of the curriculum.
• Likes to deal with big ideas; doesn't want to be bothered with details	• Prefers concepts to algorithms • Tolerates ambiguity and imprecision • Offers impulsive guesses as solutions • Uses estimation strategies spontaneously • Skims word problems first but must be encouraged to re-read for salient details • Perceives overall shape of geometric configurations at the expense of an appreciation of the individual components	• Relate manipulative models to procedures before practicing algorithms. • Reward approach as well as precise solutions. • Develop an appreciation of how much precision a situation warrants. • Reward/encourage estimation strategies as first step. • Encourage diagrams as a technique to organize data in problem solving situations. • Allow calculators to support problem solving. • Encourage multiple refinements when building geometric designs in order to incorporate all the individual parts.
• Prefers WHY to HOW	• Requires a definition of overview before dealing with exacting procedures • Requires manipulative modeling before developing a concept or algorithm • Likes to set up problems, but resists following through to a conclusion	• Offer opportunities to work in cooperative groups.
• Prefers nonsequential approaches, involving patterns and interrelationships	• Prefers successive approximations approach to formal algorithms • Addition and multiplication facts involving 9s more readily generated because of underlying patterns that are recognized but not verbalized • Not troubled by mixed practice worksheets • Comfortable with horizontal formats for long calculations • Can offer a variety of alternative answers or approaches to a single problem • Can appreciate operation needed in a word problem but has difficulty following through to an exact solution • Likes logical problem solving in the form of general reasoning problems	• Allow alternative calculation procedures. • Help students to create their own handbooks of typical problems. • Generate arithmetic facts through relationships to known facts; e.g. doubles for + facts. • Emphasize area model for multiplication. • Start with real-life situation and tease out more formal arithmetic topics. • Use simulations, relating similar concepts/approaches in a variety of different situations. • Model complex problems with similar problems in simpler forms. • Give two grades on word problem activities; one for correct approach; one for exact final solution. • Include general reasoning examples in logical problem solving activities.
• Challenged by demands for details or the requirement of precise solutions	• Difficulties with precise calculations • Difficulties offering rationale for correct solutions	• Encourage students to describe the approach or conceptual underpinning even if they cannot mobilize an exacting procedure.
• Prefers performance based or portfolio type assessment to typical tests • Prefers comprehensive exams to quizzes and unit tests • More comfortable recognizing correct solutions than generating them	• May be overwhelmed when faced with multiple examples	• Consider a variety of assessment techniques • Allow oral presentations. • Do not always require exact solution but sometimes grade homework and tests only for correct approach. • Include some multiple choice items on tests.

Table 3

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Mathematical Learning Profiles and Differentiated Teaching Strategies

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Conclusion:

The *Child/World* System allows teachers to achieve an understanding of the dynamic interplay that affects a student's learning in mathematics. It leads to the delineation of specific Mathematical Learning Profiles. Extensive clinical investigations and classroom instruction, along with rigorous research efforts, have corroborated the presence of specific Mathematical Learning Profiles. Those learning profiles involve differences in development as well as preferences for models and preferences for approaches. Complicating the consideration of learning profiles in mathematics are more general memory and language issues that intrude on efforts in mathematical activities.

The understanding of Mathematical Learning Profiles helps teachers offer specific approaches and strategies that make use of students' areas of relative strengths, that minimize areas of vulnerability and that support areas of specific deficit, ensuring the comfortable participation and growth of all students in the mathematical arena. The importance to teachers of understanding Mathematical Learning Profiles is that they lead to the development of more effective learning strategies which, in turn, allow more students to experience success in the domain of mathematics.

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