INTERDISCIPLINARY POPULATION PROJECTS IN A FIRST SEMESTER CALCULUS COURSE

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ABSTRACT: We discuss how applications from microbiology and sociology can be used to involve students and give a physical context for critical concepts in a first semester calculus course. Using data they collect themselves (from bacteria or yeast grown in a laboratory, for example), students develop elementary models of population growth in interdisciplinary class projects. Using the tools of calculus, demographic software, and available technology (CAS or graphing calculator), students then proceed from "microcosm to macrocosm," moving from biology to sociology, to address questions of human population projections.

KEYWORDS: Calculus, interdisciplinary projects, biological modeling, demographic modeling, technology-enhanced learning.

INTRODUCTION

This paper describes a series of interdisciplinary projects developed by the authors at St. Michael's College. Both of us have incorporated various projects in many of our courses over the years. One of the most successful of these projects involved growing different microorganisms during calculus courses. The student-collected data from these experiments and the resulting models became examples to illustrate the physical significance of such concepts as asymptotes, rates of change, exponential growth, areas under curves, and the importance of mathematical modeling. Encouraged by the success of this particular class project, we chose it as a basis of a case study for an integrated science course in our continuing education department. We extended the idea of exploring the population growth of organisms in the laboratory by introducing the disciplines of sociology and demography into the case study and expanding into the much more complex area of modeling human population.

After developing this material for the continuing education curriculum, we extracted the most exciting aspects to bring back into the regular calculus classroom as a series of projects in biology and demography. This article is an overview of these projects with some selected highlights. Complete versions of the entire package of projects are available. One version can be used with any CAS or graphing calculator [1], and the other depends specifically on the Maple CAS [2]. Both give complete lab procedures for all the biology experiments, web sites and recommended software for studying demography, worksheets for data collection and student project reports, graphics for in-class demonstrations, and discussion resources. Samples of these materials are included in the appendices at the end of this paper.

We continue to use the biology and technology components extensively in the classroom with excellent results. Our geography department has successfully used the demographic software and web sites for years and we are now preparing to incorporate these resources into the demographic modeling projects of our upcoming calculus courses. We continually strive to expand and improve our material and would enjoy hearing from anyone who tries these or similar projects.

OBJECTIVES AND OVERVIEW

The main objective of these projects is to give students interesting, motivating contexts for the material in a first semester calculus course. The students become possessive of and involved in their own data–*their* yeast cultures, *their* chosen country's census information. Concepts such as rate of growth, concavity, differential equations, and area then have a physical context that personally interests the students. Students also gain greater appreciation of the technological resources available to them. A student confronted with the computational tedium of manipulating real data soon realizes the value of a CAS or graphing calculator. Similarly, a student exploring the effects of changing birth and death rates in Nigeria is quickly impressed with the population resources available on the World Wide Web.

Furthermore, these projects demonstrate how a mathematical model can address complex questions and yield insights simply not discernible from the raw data alone. This strongly motivates the use of modeling and the importance of mathematics in studying biological and sociological issues (and, by extension, issues in other fields). Furthermore, in proceeding from microbiology to demography, students see the use of laboratory experiments as a microcosm for larger questions, and they experience the process of questioning and validating mathematical techniques. These experiences give them a taste and a vision of how this class is likely to be useful to them in any chosen career or endeavor.

Ideally these projects run throughout a first semester calculus course. Some lab experiments take several weeks to develop growth patterns, and these should be started at the beginning of the course. Others take as little as a few days to complete. Meaningful discussion of the data from these labs can begin after the definition of the derivative has been developed and various properties and examples of derivatives have been considered. The projects are designed to give a context for and hands-on experience with central concepts such as derivatives, exponential and logarithmic functions, properties of graphs, and integration in roughly the order that they are introduced in a standard elementary calculus course.

The biology experiments are tailored to illustrate the growth functions typically encountered in the calculus course. For example, in addition to the experiments using microorganisms, the project package ([1], [2]) includes experiments using slime molds and flour beetles which demonstrate exponential growth. Also exponential and logistic growth can be empirically studied in projects using ciliated protistans and/or yeast cultures with varying nutrient levels. Finally, for the sake of comparison, sustained growth can be modeled using pond water.

After analyzing biological growth, a major research project toward the end of the course allows students to investigate how their laboratory results can be extended to the study of human population growth. This final project involves synthesizing all the important concepts they have learned throughout the course, and then applying them in a specific field. The following scenes from the 1973 science fiction classic movie *Soylent Green* [Fleischer, R., Director. MGM] and subsequent questions are offered as motivation and to begin discussion.

> The year is 2022 AD. Forty million people crowd New York City. An air-locked plastic bubble preserves the small handful of sickly trees remaining. Bodies pack stairwells at night. Only the very wealthy can afford fresh vegetables, strawberry jam, or even hot running water. Teeming masses of desperate people mill around

> derelict cars on filthy streets, wearing surgical masks against the thick yellow smog. They fight for crumbs of government-supplied protein crackers, and bulldozers control daily food riots, scooping crying, starving people into bloody piles. Sanitation trucks soon arrive for routine collection of the dead.
>
> Is this *your* future? How would you know whether or not it is? Should you believe people who say it is your future? Why would anyone think this is a realistic model of the future? Who would predict such population growth and on what basis? If you don't believe this scenario, can you develop a better model?

This leads students to grapple with the highly complex issue of human population modeling. There are references to some papers with dramatically different population projections for students to consider at the beginning of the study and then to discuss at the end in light of what they have learned. The final project involves choosing a country or geographic region and then using a variety of library resources and electronic tools such as the internet and IntlPop (a demographic projection software package – see the list of internet sites at the end of this article) to gather data. This information is used to develop and then refine population models for the countries. Hopefully, students will discover some of the inherent difficulties of modeling human population change as they compare their results with known census reports. But as they interpolate and extrapolate based on their data and models, they will also learn a real respect for the power of mathematical modeling, and the value of the mathematical skills they learned in calculus.

CONSIDERATIONS AND RESPONSES

Almost every standard textbook includes examples of exercises using the growth of organisms, and many consider the question of human population growth (unfortunately sometimes poorly, misleadingly assuming exponential growth). However, the difference in the impact on the students between doing a few text book exercises and participating in activities is overwhelming. It obviously takes some time, energy, and resources to incorporate projects into a course, but it is well worth the investment.

Unless the person teaching the course has sufficient background in biology and demography, some interaction with faculty in other departments will probably be necessary. We had very positive responses from the colleagues we approached in various departments, and the rewarding interaction is one of the great benefits of incorporating these projects in our courses. Faculty in sociology and geography enthusiastically shared their expertise and a wealth of resources with us. They provided publications, data reports, software training, references to exciting web sites, and more.

Initially, we were hesitant to approach members of the biology department because we were afraid that our projects might be an imposition on their time and resources. We wanted observable data, but needed to avoid problems with moving our students into labs, scheduling lab times, obtaining contaminated cultures, etc. Ideally, we hoped for prepared cultures to bring in a box to our students so they could quickly take observations before or after class. We explained our projects and our needs to one colleague in biology and found that streaking out a few plates with a common microorganism was a routine procedure for a biologist. The biologist we approached was happy to provide cultures, and has graciously grown many specimens for our courses ever since. Members of the biology department provided us with the lab procedures included in the project package, and they seem genuinely enthused about having biology emphasized in our mathematics classes.

Time constraints are an important consideration when incorporating projects such as these into the curriculum. As the projects are set up, they don't take much class time. Observations can be quickly made before or after class, or the cultures can be made available for students to observe outside of class. In-class demonstrations are similar to and replace what would normally be covered in a standard lecture. The usual concepts are presented, but with emphasis on and reference to data from on-going experiments.

Projects can absorb a lot of out-of-class time, but not a prohibitive amount. Although it takes some time for the teacher to set up the experiments and to become familiar with the various resources, it isn't excessive, especially if only one new project is added to a course at a time. Evaluating the projects can be very time consuming, since each submission will be based on a different set of data. Grading can be simplified by creating a "template project" on the computer or calculator, and simply modifying the data set for each submission. Having students work in groups also significantly reduces the amount of grading.

Time constraints can be even more problematic for students. Because most of the research and analysis for these projects is done by the students outside of class, we found we had to be very aware of how much we were asking students to do in addition to the usual heavy calculus workload. Students rarely have large blocks of time in their schedules for projects such as these, and this problem can be exacerbated by the conflicting schedules of students working in groups. These projects are a lot of fun, they enliven the class, they generate great student responses, and as a result it can be very tempting to overdo them and overwhelm the students.

But... it is *because* these projects are a lot of fun, enliven the class, and generate great student responses that they are so worth doing. The activities really seem to achieve the goal of involving the students and reinforcing the concepts. Our end-of-term student evaluations testify to this. The most common response is that the projects are challenging but fun. (We were surprised and encouraged by how often the word "fun" appeared in these evaluations). Here is a sample of student comments:

> "...the 'hands-on experience' aided in my understanding of the concepts."
>
> "Instructor could explain why material was important in the real world."
>
> "What I learned in this class will definitely help me in my major."
>
> "The projects add a fun twist to the class and they help us to learn some of the concepts... it is better to learn by finding out the answer by yourself rather than someone telling you the answer."
>
> "I did not know what to expect from [calculus] in the beginning, but now I am aware of its purpose in life. The things taught in the class were things that one can apply to many aspects of life and several different occupations."
>
> "Group projects: A great idea, very good to help us to see the physical reality of calculus."

What follows are some of the specifics of these projects: a sample lab procedure (Appendix 1), one of the in-class demonstrations and some of the questions the students address (Appendix 2), and available resources (Appendix 3). The full package ([1], [2]) is available from the authors to anyone who would like more information.

APPENDIX 1: SELECTIONS FROM LABORATORY NOTES/EXERCISES

The following is one of the biological experiments that we introduced into our first semester calculus class. For information on other types of experiments, such as those involving slime molds/flour beetles (exponential growth) and ciliated protistans (exponential and logistic growth), see [1] or

[2]. It should be emphasized that to be successful these experiments need to be started during the first week of class so that students can collect data periodically during the course.

An Experiment

In this experiment bacterial cultures get restricted nutrients and exhibit logistic growth. Exponential growth will initially occur, then be restrained by the limited nutrients. The amount of time needed to see the logistic growth pattern will depend upon the incubation temperature of the agar plates. At 35°C, it could take 24 to 36 hours, whereas at room temperature it may require several days.

Equipment needed:

- Bacterial culture (*Bacillus subtilis*)
- Tryptic soy agar plate (DIFCO)
- Ruler

Procedure:

Streak out *Bacillus subtilis* on a tryptic soy agar plate (DIFCO) so that you obtain well separated colonies. To accurately measure colony growth, it may be most effective to grow a single culture on a single plate. The plates can be incubated at either room temperature or 35°C, depending upon how quickly you want growth. At room temperature, observations should be made about once a day. The diameters of the resulting bacterial colonies can be measured with a finely marked ruler.

Data Collection

Students periodically record their data in a table such as the one below:

Name: ____________________
Experiment: ____________________
Organism observed: ____________________
Measurement method: ____________________

Observation #	Date and Time	Number of Hours (in decimals)	Observation

Modeling

In class, we discuss with students the creation and properties of some basic population models, including the exponential and logistic models. With these examples in mind, students use available technology (CAS or graphing calculator) to create a mathematical model that fits their experimental data and write a report of their results. They include the following in their laboratory reports:

1. A graph of data points
2. Work showing how the constants for the modeling function were determined
3. A comparison between the actual data and the values predicted by the modeling function
4. A plot of the modeling function
5. A plot of the function with the actual data points plotted on the same graph

Note: In each experiment we are a priori assuming that the data is modeled by a certain growth curve. We briefly mention factors affecting the choice of curve and similar curve-fitting issues to our students as they arise. However, there usually isn't room in our first semester calculus syllabus for an in-depth discussion of curve-fitting methods. Thus, students use "eyeball" estimates to obtain the best curve they can. They may try several different values for their model constants (for example, adjusting the carrying capacity in the logistic model) or try different data points to solve for the growth constants. In a calculus course which does cover such techniques as the least squares method, these labs provide an ideal opportunity to discuss and reinforce those methods.

Analysis

Having created a mathematical model, students are then asked questions that cannot be answered using their raw data only. These questions help students connect their experiments with calculus concepts discussed in class (such as the derivative of a quantity representing its instantaneous rate of change at a particular time). Whenever appropriate, students include calculations, comments, and units with their answers. Examples of such questions are:

1. What are the constants you used in creating your model? What physical properties do these constants represent?

2. What does your model give as an estimate of the area covered by your colony at 10:00 AM on the fifth day of the experiment?

3. At what rate was the area increasing at that time? What does this number mean?

4. Was the rate at 10:00 AM on the fifth day greater or less than the rate at 10:00 AM on the eighth day? What does this say about how your colony is growing?

5. When did the area stop growing at an increasing rate and start growing at a decreasing rate? What causes this?

6. What was the average area covered by your colony between the first and last measurement?

7. Is there a time when the actual area is equal to the average area? If so, when?

8. What do you think the population would be 24 hours after your last observation? 48 hours later? How long do you think your model would be accurate?

9. Comment on your results. How well do they seem to reflect or predict reality?

APPENDIX 2: AN IN-CLASS DEMONSTRATION

To help students understand some of the concepts discussed in class, we use in-class demonstrations. The in-class demonstration below introduces students to logistic modeling. Ideally, this demonstration would be replicated using whatever technology (CAS or graphing calculator) the students are using. However, demonstrations can also simply be reproduced on overheads and discussed in class. The full Maple worksheet with code for this and other demonstrations can be found in [2]. The demonstration below enables us to discuss the following mathematical concepts in a biological context:

- Review of concavity and inflection points of curves as well as horizontal asymptotes of graphs

- Introduction of the logistic curve and its mathematical properties, including the significance of its concavity changes and horizontal asymptote (called the *carrying capacity* of the environment)

- Illustration of the characteristics of logistic growth with some real world examples (such as disease epidemics)

- Comparison of *exponential growth* (growth rate proportional to the population size), *limited growth* (growth rate proportional to distance between the size of the population and the carrying capacity), and *logistic growth* (a combination of exponential and limited growth with the growth rate proportional to the size of the population and the distance between the size of the population and the carrying capacity)

- Construction of a logistic curve model using some data points to compute curve parameters and verification of the accuracy of the resulting model with other data points

- Introduction of the notions of an integral and the average value of a function (to measure average population size)

Modeling with the Logistic Curve

This in-class demonstration should follow a discussion of various population models (exponential, restricted, and logistic growth) and the differential equations satisfied by each. The parameters below are found by "eyeball" estimates. However, as mentioned in Appendix 1, curve-fitting and sensitivity to parameter values can be addressed more rigorously if appropriate to the course.

In this example, a logistic curve is used to model the growth of a bacterial culture. The culture was grown in a petri dish, and every few days students measured the diameter of an isolated colony. Also, initial measurements were only accurate to two decimal places. The results in the demonstration showing several decimal places reflect not accuracy but the default setting on the computer or calculator used to generate them. This should be discussed with the students.

Below is data collected from a bacteria culture during the spring semester. Here are the recorded data points: t_i gives the time and d_i the diameter (in millimeters). The times are the number of hours (converted to decimals) after 12:45 PM on 4/11. The areas are given by a_i and computed by $a_i = \frac{\pi d_i^2}{4}$.

t_i	d_i	a_i
$t_0 = 0$	$d_0 = 2.5$	$a_0 = 1.5625\pi = 4.908738522$
$t_1 = 23.16666667$	$d_1 = 5$	$a_1 = 6.25\pi = 19.63495409$
$t_2 = 49.9167$	$d_2 = 5$	$a_2 = 6.25\pi = 19.63495409$
$t_3 = 98.5$	$d_3 = 6$	$a_3 = 9.00\pi = 28.27433389$
$t_4 = 148.85$	$d_4 = 6.6$	$a_4 = 10.8900\pi = 34.21194400$
$t_5 = 167.5$	$d_5 = 6.75$	$a_5 = 11.390625\pi = 35.78470382$
$t_6 = 192.1667$	$d_6 = 6.52$	$a_6 = 10.627600\pi = 33.38759009$

Then, pairs of data points, together with an estimate for the carrying capacity M, are used to determine a logistic function $L(t)$ which models the growth of the bacteria.

In general, a logistic curve has the following form, for some constants M, B, and k:

$$L(t) = \frac{M}{1 + Be^{(-Mkt)}}.$$

The constant M is the carrying capacity, or limit of what the system can support. A quick way to estimate M is to plot the data points and do a freehand sketch through them to estimate M. Figure 1 shows a plot of the points.

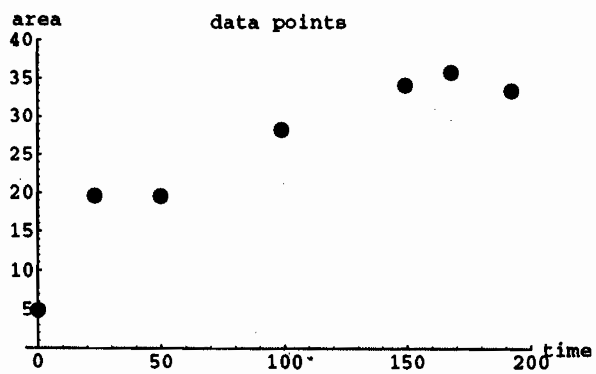

Figure 1. Plot of data points for bacteria culture.

That second data point looks a little too big, and the last one a little too small. This is probably due to inaccuracy in measurement. This unfortunately happens, but we should still be able to get a good model. A good guess for the carrying capacity, M, seems to be 34.5, so

$$L(t) = \frac{34.5}{1 + Be^{(-34.5kt)}}.$$

Now, use t_0 and a_0 to solve for B, getting that $B = 6.028282287$. Thus, $L(t)$ looks like

$$L(t) = \frac{34.5}{1 + 6.028282287e^{(-34.5kt)}}.$$

Next, use t_2 and a_2 to find k. Normally, we'd use t_1 and a_1 here, but as we noted above, that data point appears inaccurate, so we go to the next one that looks good. We find that $k = 0.001204767896$.

With this, $L(t)$ looks like

$$L(t) = \frac{34.5}{1 + 6.028282287e^{(-0.0415649241t)}}.$$

Now, check how well this function models the remaining four data points. Here is what the function $L(t)$ gives at the times t_0 through t_6:

t_i	$L(t_i)$	a_i
t_0	4.908738522	4.908738522
t_1	10.44986633	19.63495409
t_2	19.63495409	19.63495409
t_3	31.34950928	28.27433389
t_4	34.07758738	34.21194400
t_5	34.30413399	35.78470382
t_6	34.42948532	33.38759009

All but the second look pretty good.

Figure 2 shows the graph of $L(t)$ with the actual data points superimposed on it.

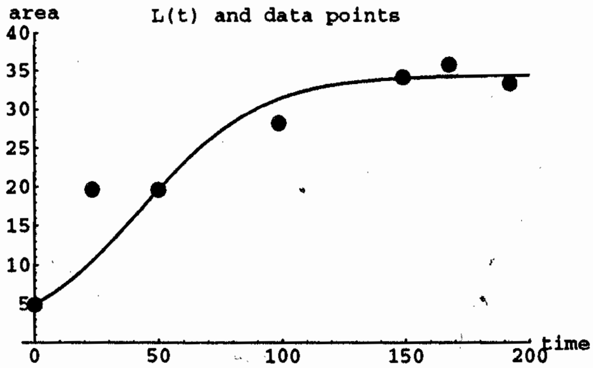

Figure 2. Plot of data points and model for bacteria culture.

Analyzing the Results

With this mathematical model, we can answer some questions we couldn't have answered just using the observed data:

1. What was the area covered by the colony at 10:00 AM on 4/16?

   Answer: First, determine the value of t at that time by counting 5 days less 2 $\frac{3}{4}$ hours from the start of the experiment at 12:45 on 4/11. This gives

   $$t_s = 5(24) - 2.75 \qquad \text{or} \qquad t_s = 117.25.$$

   Now, evaluate $L(t)$ at that time to find the area covered:

   $$L(t_s) = 32.97968560.$$

2. At what rate is the area increasing at that time?

   Answer: We need the derivative of $L(t)$ to answer this. Fortunately, $L(t)$ satisfies the differential equation $y' = ky(M - y)$, so we have the following formula for $L'(t)$:

   $$L'(t) = kL(t)(M - L(t)).$$

   Evaluate that at t_s to get the rate:

   $$L'(t_s) = .06040644899.$$

3. What is the carrying capacity?

   Answer: The carrying capacity is just $M = 34.5$.

4. When does the rate slow down? In other words, when does the population stop growing at an increasing rate and start growing at a decreasing rate? (This is just asking where the inflection point is).

   Answer: You can get a good estimate for this just by looking at the graph. It looks like about 43 hours after the start of the experiment. Alternatively, you can find the second derivative of $L(t)$, set it equal to zero and solve for t, getting that the t-coordinate of the inflection point is 43.22107658.

5. What is the average area covered by the colony during the course of the experiment?

   Answer: First, we need to find the total area by integrating $L(t)$ from t_0 to t_6, which gives 5012.92708. Dividing this by $t_6 - t_0 = t_6$ gives that the average area covered by the colony during the course of the observed experiment is 26.08635080 mm^2.

6. Is there a time when the value of the function $L(t)$ is equal to its average value? If so, find it. Is there a theorem which tells you ahead of time that there must be such a time?

   Answer: The mean value theorem for integrals says that there must be a solution to $L(t) = 26.08635080$ in the interval $[t_0, t_6]$. Solving this equation yields that $t = 70.44520135$.

APPENDIX 3: A RESOURCE LIST

Biological Supplies

Many materials for the biological experiments can be found at a local grocery store. Either of the following biological supply companies can provide everything else:

Carolina Biological Supply Co.
2700 York Road
Burlington NC 27215 USA

Connecticut Valley Biological
82 Valley Road, P. O. Box 326
Southampton MA 01073 USA

Demographic Discussion Material

The following texts provide good background for issues concerning human population growth and projections:

Alonso, William and Starr, Paul (editors). 1987. *The Politics of Numbers*. New York: The Russell Sage Foundation.

Cohen, Joel E. 1995. *How Many People Can the Earth Support?* New York, London: W. W. Norton Company.

Ehrlich, Paul R. 1968. *The Population Bomb*. New York: Ballantine Books.

Fleischer, Richard, Dir. 1973. *Soylent Green*. With Charleton Heston, Leigh Taylor-Young, Chuck Connors, Joseph Cotton, Edward G. Robinson. MGM. Available on videocassette.

Harrison, Harry. 1966. *Make Room! Make Room!* Boston: Gregg Press.

Haub, Carl. December 1987. Understanding Population Projections. *Population Bulletin*. 42(4): 3-42.

Population Reference Bureau. 1997. *1997 World Population Data Sheet*.

Shryock, Henry S., Jacob S. Siegel, and Associates. 1973. *The Methods and Materials of Demography (Volume 2)*. Washington DC: U. S. Bureau of the Census.

Simon, Julian L. 1990. *Population Matters*. New Brunswick: Transaction Publishers.

Tapinos, Georges and Phyllis T. Piotrow, 1980. *Six Billion People*. New York: McGraw-Hill Book Company.

Internet Sites

The following sites provide useful, current information concerning human population issues:

`http://geosim.cs.vt.edu/` Home page of Project GeoSim at Virginia Tech (source for IntlPop)

Project GeoSim is a research project of the Departments of Geography and Computer Science at Virginia Tech. Two modules created through this project include HumPop, a multimedia tutorial population program, and IntlPop, a population simulation program for the various countries/regions around the world. These modules can be downloaded from this site.

`http://www.census.gov/` Home page of the US Census Bureau

Provided by the U. S. Census Bureau, this site offers a rich collection of social, demographic, and economic information as well as 1990 census data for various locations in the United States.

`http://www.nidi.nl/links/nidi6000.html` Home page of NiDi

Maintained by the Netherlands Interdisciplinary Demographic Institute, this site offers a comprehensive overview (with over 370 external links) of demographic resources on the internet.

`http://www.popnet.org/` Home page of Popnet

Created by the Population Reference Bureau, Popnet offers links to many sites containing global population information. It contains a clickable world map offering a directory of websites for various regions around the world.

`http://www.prb.org/` Home page of Population Reference Bureau (PRB)

The Population Reference Bureau is a nonprofit, nonadvocacy organization which is "dedicated to providing timely, objective information on U. S. and international population trends".

`http://www.iisd.ca/linkages/` Home page of Linkages

Provided by the International Institute for Sustainable Development (IISD), Linkages is "designed to be an electronic clearing-house for information on past and upcoming international meetings related to environment and development.

http://popindex.princeton.edu/ Home page of the Population Index

Created by the Office of Population Research at Princeton University, the Population Index presents an "annotated bibliography of recently published books, journal articles, working papers, and other materials on population topics".

http://coombs.anu.edu.au/ResFacilities/DemographyPage.html
Home page of Demography and Population Studies at ANU

Provided by the Coombs Computing Unit at the Australian National University, this site contains over 150 links to useful demographic information centers and facilities world-wide.

Addresses of internet sites change frequently, as do the sites themselves. If any of the above sites become invalid, many similar sites will still be available and can be found using a web browser. To access the most current demographic information, consult a current almanac or recent edition of Statistical Abstracts.

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REFERENCES

Our entire package of projects is contained in both of the following manuscripts. If you would like more information, please contact either of the authors.

1. *How Many People are in Your Future? Elementary Models of Population Growth*; to appear with other integrated science case studies written by professors from various disciplines at St. Michael's College in a compilation to be published by McGraw-Hill Primis. The material with this version can be used with any available technology.

2. *Microcosm to Macrocosm: Population Models in Biology and Demography*; preprint available on request. In this version, the in-class demonstrations and laboratory exercises/solutions are written in Maple.

BIOGRAPHICAL SKETCHES

George Ashline received his BS from St. Lawrence University, his MS from the University of Notre Dame, and his PhD from the University of Notre Dame in 1994 in value distribution theory. He has taught at St. Michael's College for several years. He is a participant in Project NExT, a program created for new or recent PhD's in the mathematical sciences who are interested in improving the teaching and learning of undergraduate mathematics.

Joanna Ellis-Monaghan received her BA from Bennington College, her MS from the University of Vermont, and her PhD from the University of North Carolina at Chaple Hill in 1995 in algebraic combinatorics. She has taught at Bennington College, at the University of Vermont, and at St. Michael's College since 1992. She is a great proponent of active learning and has developed materials, projects, and activities to augment a variety of courses.