

## A House of Your Own: The Mathematics of the American Dream (Part II)

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### Overview

**Description:** This is a lesson plan for a project wherein students choose the most appropriate mortgage program to finance a new home, using materials drawn from community resources.

**Keywords:** House buying, home mortgages, simple interest amortized loans, amortization schedule, financial mathematics.

**Project Goals:** To engage students in a hands-on experience with some of the calculations and analysis involved in buying a home; to provide motivation for various topics in financial mathematics; to convince students of the relevance of mathematics in their daily lives.

**Learning Objectives:** Students will be able to apply financial formulas; compile amortization schedules; use critical thinking skills to compare mortgage programs and justify a choice of programs; make appropriate use of technology; reinforce basic algebra skills; and work cooperatively in small groups.

**Prerequisites:** Organizational ability, attention to detail, comfort with basic algebraic concepts such as order of operations and properties of exponents, and exposure to elementary financial formulas.

**Course level:** Any course, such as Algebra II, Pre-calculus, or Financial Mathematics, in which students have or develop an understanding of basic algebraic methods, or any course that develops basic formulas for simple and compound interest and amortized loans. This is a good capstone project for the financial component of such a course.

**Technology used:** At least a calculator with a memory function and exponentiation key, and access to an amortization schedule generator,

eg. on the Internet. Spreadsheet software may be used.

**Required equipment/materials:** Area real estate listings, current mortgage rates, local property tax information.

**Time estimates:** *Instructor preparation:* Minimal for project provided here, 2 to 6 hours for a customized project using local information. *Class presentation:* Minimal in a class that already includes basic finance formulas, several class periods to develop the formulas as ancillary material. *Student work outside of class:* About 8 to 14 hours. *Fielding questions:* Average of about 30 minutes per group. *Grading:* Roughly 20 minutes per project submitted.

### NCTM standards addressed:

#### *Content standards:*

*number and operations* – develop fluency with order of operations and exponentiation, compare monetary quantities, judge reasonableness of numeric computations carried out by technology;

*algebra* – understand relations and functions, symbolically represent and solve financial equations, interpret relationships among variables.

#### *Process standards:*

*problem solving* – solve problems arising from a specific life-skills context;

*communication* – organize information, communicate coherently with peers in a cooperative group, develop mathematics exposition through the final presentation of analysis;

*connections* – recognize and apply mathematics in a financial context;

*representation* – create and use representations to organize and communicate analysis.

## Project Description

Surely everyone has some corner of his or her heart reserved for a dream home. It may be no more than a vague notion of someday having a "place of one's own." It may be a fully imagined three-bedroom ranch, with even the carpets and countertops already selected. But, to become a reality, this dream home will likely cost thousands, even *hundreds* of thousands, of dollars. For a \$120,000 home, a borrower will pay more than \$360,000 over the life of thirty-year mortgage at 9.5%. Then there are taxes, mortgage insurance, points, fees, and more. Once students realize this, it is a short step to convincing them that it is worth studying the mathematics involved in buying a house. Even adult learners, (perhaps *especially* adult learners) who have been through the process of buying a house already are eager to learn this material.

This article includes a classroom-ready project for experiencing some of the computations and analysis involved in choosing a fixed-rate home mortgage. We collected information from area banks, real estate guides, and the town office to make the project as realistic as possible. This is a substantial project which requires students to demonstrate thorough mastery of basic financial concepts and formulas, proficiency with a calculator or software program, and analytic ability in a real-life context.

Although the house advertised in the project comes from our local area, it is easy to modify the project for any location or price range. A house/condominium/apartment can be found in a local real estate guide, current mortgage rates can be found at any bank, and tax information can be obtained through a telephone call to the town office. If a spreadsheet is used to compute the solutions, these changes, as well as yearly updates, are easy to implement since only the raw data needs to be entered in the appropriate cells to generate answers for the whole project.

The mathematics of house buying are substantial, but appropriate to any course requiring basic algebra skills. For this project, students need to know how to determine the down payment of a loan, calculate the monthly payment of a simple interest amortized loan, and use the APR to estimate prepaid finance charges. The respective equations for this are:

1.  $A = p(C - D) + D$ , where  $A$  is the amount of cash available for points and down payment,  $p$  is the points, and  $C$  is the cost of the house;

2.  $P(1+i)^n = \text{payment} \frac{(1+i)^n - 1}{i}$ , where  $P$  is the principal,  $i$  is the periodic rate, and  $n$  is the number of payments; and

3.  $(P - \text{fees})(1+i)^n = \text{payment} \frac{(1+i)^n - 1}{i}$ , where

$P$  is the principal,  $i$  is the annual percentage rate,  $n$  is the number of payments, and  $\text{payment}$  is the amount of the monthly principal and interest payments. In this equation (3), solving for  $\text{fees}$  gives an estimate of the prepaid finance charges. This is a minor part of the project and can easily be omitted.

Other mathematical activities involved in this house buying project include calculating points, understanding maximum LTV (Loan to Value) and its relation to a minimum down payment, becoming familiar with escrow of mortgage insurance and property taxes, computing property taxes from the fair market value of a home, knowing when to use the APR and when to use the rate, and preparing a long term amortization schedule.

In our own courses, most students simply use calculators for all parts of the project except the amortization schedule, which they prepare using a software package. Students can create an abbreviated (3 to 6 month) amortization schedule using a calculator. They often gain a better understanding of the mechanics of a mortgage (realizing why it is called a *simple interest* amortized loan, for example) by working out these details. However, the tedium of computing a lengthy

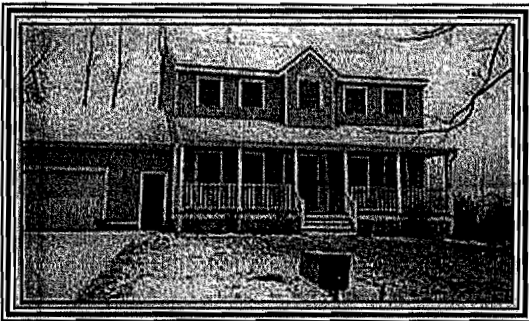
schedule by hand quickly provides great motivation for learning to use a spreadsheet or other software. In fact, although this project can be done using only a calculator, it is especially suited for courses which incorporate some kind of spreadsheet program.

Student response to this project in our courses has been overwhelmingly positive. They are motivated to do substantial mathematics and feel that the experience is valuable. Their enthusiasm stems from being convinced that what they have learned will help them realize their dream of someday having a home of their own.

The classroom materials and solutions for this project are provided.

### The House Buying Project

You have decided to buy the house advertised below. The house is located in Stoneville, where property appraisals were redone fairly recently, so that properties are appraised at 86% of market value. The tax rate in Stoneville is \$1.93.



#### Idyllic Retreat

Rural Stoneville, convenient to mountains and lake. 5.4 acres with beautiful pond and stream. 2,500 SF, 3BRs, 2-1/2 baths. Finished basement, studio/office. Chef's kitchen. Motivated sellers. \$129,500.

You have saved \$12,000. \$3,500 of that amount is needed to pay for prorations, taxes, lawyer's and inspector's fees, appraisal fee, utilities hook ups, mortgage insurance prepayment, and other charges. The remainder of your savings will pay for your down payment and any points.

Since you will have to borrow more than 80% of the value of the house, you will probably have to pay mortgage insurance. Assume this costs \$75 per month. In addition, you will need to use the local tax rate and percent of fair market value for the town to estimate your yearly property tax. Divide this amount by twelve to find your monthly property tax escrow payment.

You have gone over your family's finances and can afford a maximum of \$1,100 per month in total house payments. The total monthly payment includes principal and interest on your loan, mortgage insurance, and property tax escrow.

You go to GoodCo Credit Union and get the following list of their current mortgage programs.

#### MORTGAGE RATES FIXED RATE PRIMARY RESIDENCE February 11, 1999

TERM/ TYPE	APR	RATE	POINTS	MAX LTV	Max loan amount
30 yr fixed	6.822	6.625	2.00	95 %	\$227,150
30 yr fixed	6.974	6.875	1.00	95 %	\$227,150
30 yr fixed	7.175	7.175	0.00	95 %	\$227,150
TERM/ TYPE	APR	RATE	POINTS	MAX LTV	Max loan amount
20 yr fixed	6.750	6.375	2.00	95 %	\$227,150
20 yr fixed	6.753	6.625	1.00	95 %	\$227,150
20 yr fixed	7.000	7.000	0.00	95 %	\$227,150

The six options you should consider are 30 and 20 year loans, at 2, 1, and 0 points. Your assignment is first to determine which of the six home loan programs you can afford, and then choose which of those is the least expensive. For some loans, you may not be able to afford both the points and the minimum down payment. For others, the total monthly payment may be more than you can afford. For each of the loans which you can afford, determine the total cost of the loan and then choose the least expensive. The total cost of the loan is the total interest on the loan plus the points.

Once you choose the best mortgage program, you can use the APR to estimate any prepaid finance charges. (Be sure you understand when to use the APR and when to use the rate of the loan program.) Based on your calculations, determine if GoodCo Credit Union charges any fees besides the points. For the program you have found to be the least expensive, prepare an amortization schedule for the first and last twenty-four months of the loan.

### Some Hints for the Project

1. The points are a percentage of the loan amount that you have to pay the credit union at the time you get your loan. For example, if the credit union charges 2.00 percentage points on a loan, and you borrow \$2,000, then you have to pay the credit union  $.02 \times \$2,000 = \$40$  when you get your money. This is *in addition* to the usual interest you have to pay.
2. All the loan programs in the mortgage chart are simple interest amortized loans.
3. One column in the mortgage chart is headed Maximum LTV (Loan to Value). All this means is that the credit union won't lend you more than 95% of what the house costs, so

in all cases you must make a down payment of at least 5%.

4. Remember that the more money you put down, the less money you have to borrow. The less money you borrow, the less interest you have to pay. Thus, in each of the six options, you want to make as large of a down payment as possible. However, you only have a fixed amount saved to cover both the down payment and the points. The points are computed on the amount you borrow, namely  $C - D$ , where  $C$  is the sale price of the house and  $D$  is the amount of the down payment. This means that you can find the down payment amount by solving for  $D$  in the equation  $A = p(C - D) + D$ . Here,  $A$  is the amount of money you have for points and down payment and  $p$  is the points written in decimal form (for example, for 3 points,  $p$  would be 0.03). Use this equation with the different point amounts to find the largest possible down payment in each case. Remember though that the down payment must always be at least 5% of the cost of the house.
5. To estimate the taxable value of your property, multiply the price of your house by the percent of fair market value on houses in the town. The tax rate is usually given as a dollar amount, for example \$1.25, which means that you pay \$1.25 for every \$100 of property value you own. Thus, you can think of the tax rate as a percent—1.25%. Then, to estimate your yearly taxes, multiply the taxable value of your property by the tax rate as a percent. Finally, divide the result by 12 to find the amount of your monthly escrow tax payments.

### Student Worksheet: Organize the Information

Use a copy of the following table to list your results for each loan program. If any loan program is unusable, give a reason. For example, you may not be able to afford both the down payment and the points, or the total monthly payments. Once you determine a program is too expensive, you need not finish the computations for that program. Remember that the total monthly payment is the monthly principal and interest payment plus the monthly tax escrow plus the mortgage insurance payment. Also, the total cost of the loan means the total interest plus the cost of any points.

Number of years	
Points	
Rate	
APR	
Number of months	
Down payment	
Amount of loan	
Cost of points	
Monthly principal and interest payment	
Total monthly payment	
Total interest	
Total cost of loan	

After deciding which program is both affordable and least expensive, use the table below to organize the information for that program.

<b>Cost of house</b>	
Total cash on hand	
Amount needed for closing costs other than points	
Amount left for points and down payment	
5% minimum down payment	
Taxable value of your property	
Yearly property taxes	
Monthly escrow tax payments	
Which program you decided was the best (number of years/points)	
Rate for your program	
APR for your program	
Estimated prepaid finance charges for your program	
How much you paid in points for your program	

Also, determine if GoodCo Credit Union includes any fees besides the points in your chosen loan program. Then, produce an amortization schedule for the first and last twenty-four months of your chosen program.

## HOUSE BUYING PROJECT SOLUTIONS

### Initial Cash Assessment

total cash available	\$12,000.00	maximum total monthly payments	\$1,100.00
cost of house	\$129,500.00	amount left for points and down payment	\$8,500.00
5% min down paymt	\$6,475.00	non-prepaid closing costs	\$3,500.00

### Taxes and Mortgage Insurance

tax rate	0.0193	fair market value	0.86
mortgage insurance	\$75.00	yearly tax	\$2,149.44
		monthly tax escrow	\$179.12

### Computational Procedures for the Loan Programs Below

number of months	12 times the number of years
down payment	computed by solving for $D$ in the formula $A = p(C - D) + D$
amount of loan	sale price of house minus the down payment
cost of points	points (written as a decimal) times the amount of loan
monthly P&I payments	computed using the simple interest amortized loan formula
total monthly pymts	monthly P&I payments plus monthly taxes plus mortgage insurance
total interest	number of months times monthly P&I payments, minus loan amount
total cost of loan	total interest plus cost of any points

### Values for Each Loan Program

30 years, 2 points		30 years, 1 point		30 years, 0 points	
number of years	30	number of years	30	number of years	30
points	0.02	points	0.01	points	0
rate	6.625%	rate	6.875%	rate	7.175%
APR	6.822%	APR	6.974%	APR	7.175%
number of months	360	number of months	360	number of months	360
down payment	\$6,030.61	down payment	\$7,277.78	down payment	\$8,500.00
amount of loan	\$123,469.39	amount of loan	\$122,222.22	amount of loan	\$121,000.00
cost of points	\$2,469.39	cost of points	\$1,222.22	cost of points	\$0.00
monthly payments	\$790.59	monthly payments	\$802.91	monthly payments	\$819.29
total monthly pymts	\$1,044.71	total monthly pymts	\$1,057.03	total monthly pymts	\$1,073.41
total interest	\$161,142.30	total interest	\$166,826.46	total interest	\$173,943.30
total cost of loan	\$163,611.69	total cost of loan	\$168,048.68	total cost of loan	\$173,943.30

20 years, 2 points		20 years, 1 point		20 years, 0 points	
number of years	20	number of years	20	number of years	20
points	0.02	points	0.01	points	0
rate	6.375%	rate	6.625%	rate	7.000%
APR	6.750%	APR	6.753%	APR	7.000%
number of months	240	number of months	240	number of months	240
down payment	\$6,030.61	down payment	\$7,277.78	down payment	\$8,500.00
amount of loan	\$123,469.39	amount of loan	\$122,222.22	amount of loan	\$121,000.00
cost of points	\$2,469.39	cost of points	\$1,222.22	cost of points	\$0.00
monthly payments	\$911.49	monthly payments	\$920.27	monthly payments	\$938.11
total monthly pymts	\$1,165.61	total monthly pymts	\$1,174.39	total monthly pymts	\$1,192.23
total interest	\$95,288.43	total interest	\$98,643.24	total interest	\$104,146.81
total cost of loan	\$97,757.82	total cost of loan	\$99,865.46	total cost of loan	\$104,146.81

### Analysis

Note that paying two points does not leave enough money for the minimum down payment of \$6,475. Furthermore, the total monthly payments on all of the 20 year loans is higher than \$1,100. Thus, the only possible loans are the 30 year mortgages with either 1 or 0 points. Of the two, the 1 point program has the lower total cost, and hence is the best long term mortgage. The best program which is affordable is a 30 year loan with one point.

### Estimating Prepaid Finance Charges

estimated prepaid finance charges	\$1,220.91
difference between estimated charges and cost of points	-\$1.31

Thus, there are no prepaid finance charges other than the point.

### Amortization Schedule for Selected Months

Month Number	Principal Portion	Interest	Monthly Payment	Amount Due After Payment
0	---	---	---	\$122,222.22
1	\$102.68	\$700.23	\$802.91	\$122,119.54
:	:	:	:	:
24	\$117.10	\$685.81	\$802.91	\$119,588.47
:	:	:	:	:
337	\$700.04	\$102.87	\$802.91	\$17,255.82
:	:	:	:	:
360	\$798.34	\$4.57	\$802.91	\$0.00

## Recent Developments on the Mathematics PRAXIS Exam and Suggestions for All Concerned Parties

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### Introduction

The authors of this article are a mathematics teacher educator and five prospective mathematics teachers. The article was written by the mathematics teacher educator with research assistance, insights, and editorial clarifications provided by the prospective teachers. We wrote this article to raise awareness concerning the PRAXIS II Subject Assessment that is required of prospective secondary teachers seeking certification in New Jersey. In particular, we outline our concerns to changes made in the passing score for the mathematics test in the year 2004. We also make suggestions designed to address some of these concerns in the hope that all participants involved in this process (state and testing agencies, teacher education institutions, and teacher candidates themselves) will be better informed about developments involving the PRAXIS.

### The PRAXIS II Exam: Background and Recent Changes

The PRAXIS II Exam for Mathematics Content Knowledge is given by the Educational Testing Service (ETS). It is used by 26 states and the District of Columbia as a measure of mathematics knowledge and as one criterion in obtaining state teacher certification. The exam is given several times during the school year (in September, November, January, March, April and June) and students typically receive their scores four to six weeks after taking the exam.

In 2003, under Education Commissioner William Librera, concerns were raised about New Jersey "allowing among the lowest passing scores on national teacher exams" (Mooney, 2003, p.17). Our own analysis of scores in the booklet *Understanding Your PRAXIS Scores 2003-2004* confirmed this. For example,

looking at the sample whose scores were reported (D.C. and the 21 states Alaska, Arizona, Connecticut, Georgia, Hawaii, Indiana, Kentucky, Maryland, Minnesota, Mississippi, Missouri, New Hampshire, New Jersey, Ohio, Oregon, Pennsylvania, South Carolina, Tennessee, Vermont, Virginia, and West Virginia), New Jersey's passing math score of 130 is in the lowest quartile (ETS, 2003). The average passing score for this sample was 134.7 with a range from 116 to 147. Of the five state scores not reported, we discovered that North Carolina requires a minimum combined score on the Content Exam (10061) and Pedagogy Exam (30065). Kansas had not set qualifying scores (email communication with Teacher Education and Licensure, Kansas State Department of Education). The three passing scores that we did obtain (Colorado, 156; Idaho, 141; Nevada, 144) only placed New Jersey lower in the ranking.

In an attempt to raise New Jersey's passing score into the top quartile, Commissioner Librera recommended an increase in PRAXIS passing scores to be implemented in phases (New Jersey Board of Education Meeting Highlights, 2004). In June of 2004, the New Jersey State Board initiated the first phase by adopting a resolution raising the passing scores needed to obtain teaching certificates (State Board Highlights, 2004). Effective September 1, 2004, the passing math score was raised from 130 to 137.

### Our Experience and Concerns

For teacher candidates not yet certified to teach by September 1, 2004, the old passing score of 130 became invalid. For example, imagine that a candidate passed the June 2004 exam with a score between 130 and 136 inclusive. Imagine also that the candidate had



not completed other requirements for certification (like student teaching or coursework). If the candidate had not applied for certification before September 2004, then a score of 130 to 136 would not meet the requirements for teacher certification.

Our experience with these events was less than positive. Despite all the activity surrounding this development, we heard nothing of this news until the fall of 2004. As the professor of our fall 2004 Methods class, the university asked me to relay the news about the change in passing scores to my students. Well into their fall semester of classes (with class and work schedules in place), those candidates affected by the news could do nothing but take on the unplanned responsibility of preparing again for an exam they thought was behind them. With some candidates taking upwards of three classes, working, and doing field experiences, this burden became, well, just that—a burden.

In fairness to Commissioner Librera and the NJ Board of Education, additional regulations were effected in September that provided teacher candidates some flexibility in using GPA and PRAXIS scores to apply for certification. For example, a minimum GPA of 2.5 (the state-required GPA is 2.75) would be considered to meet requirements for certification if a candidate scored 10 percent or higher above the PRAXIS cutoff score of 137. Of course, this option cannot help people who did not obtain the minimum 137. Alternatively, a candidate who fell short of the 137 by 5 percent or less could still meet requirements if his or her GPA was 3.5 or better. But as some candidates in our methods classes (with GPA's of 3.5 or better) noted, "Wouldn't a school be skeptical if I obtained certification without the necessary 137 on my PRAXIS exam?" As a result of this reasoning and these circumstances, this option too became a non-option.

We also had questions. Why weren't test-takers notified of the change in passing score by ETS or by the NJ Board of Education?

In particular, why weren't test-takers notified during the summer of 2004 so that they could plan fall schedules accordingly and have the option of studying during the summer? What about a grandfather clause allowing passing scores that were in place prior to September 2004 to apply toward certification obtained within, say, one year? Given the possibility of scores rising again, when is the optimal time to take the test? All of these questions were left unanswered by ETS and the NJ Board of Education, and many of the prospective teachers in my class (even those who scored 137 or better) were left with the feeling that they had been treated unfairly and unprofessionally.

### Some Suggestions

Although we realize there are some things we cannot change, we do have some suggestions for ETS, the NJ Board of Education, teacher preparation institutions, and teacher candidates preparing to take the PRAXIS.

First: As the institution administering the exam, ETS has a list of all candidates who took the test and who could be affected by changes in state policy. If a state board of education enacts a policy that affects the outcome of test-takers, ETS and that board could collaborate to inform the affected parties. Communicating through test-takers' email addresses provides a low-cost alternative to mailings. And a timely announcement about the changes could enable prospective teachers to plan accordingly.

Second: Teacher preparation institutions need to be aware of these developments and advise students accordingly. They could provide a forum for ensuring that their teacher candidates are informed of state policies. Also, given the current state of affairs, it does not make sense for a candidate who has two years worth of work left for certification to take the PRAXIS since (without a grandfather clause) the scores may change by the time he or she applies for certification. A candidate who has

the means can try his or her hand at the exam, but there is no point in taking 2 or 3 exams to pass a given year's cutoff score when that score may not be in effect at the time of certification. Some caution, good advice, and good planning are clearly necessary (and can save prospective teachers time and money!).

Third: Teacher candidates need to stay abreast of developments that may affect them and use the information to raise questions and obtain answers. Some websites that can provide important updates include [www.state.nj.us/njded/news](http://www.state.nj.us/njded/news) (NJ Department of Education News) and [www.njasa.net/bd\\_issues/meeting\\_highlights](http://www.njasa.net/bd_issues/meeting_highlights) (NJ State Board of Education Meeting Highlights). Teacher candidates also need to be aware of their institution's requirements for program completion and certification in comparison to state options provided. Being informed is part of a prospective educator's responsibility.

### Concluding Remarks

Attracting *qualified professionals* to teaching means more than ensuring the candidate's *qualifications*. It means treating others, and being treated, as *professionals*. We hope the suggestions above serve to initiate teacher candidates into teaching in the open, respectful, and informed manner that is becoming of their profession.

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