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HOME SWEET HOME: A FINANCIAL INCENTIVE FOR THE LOWER LEVEL MATHEMATICS COURSE

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ABSTRACT: We describe a house-buying active learning project which is designed to motivate students in lower level college mathematics courses. In addition, we discuss the mathematical content, implementation, evaluation, and benefits of the project. The project involves many aspects of house buying, such as choosing a mortgage program subject to financial constraints, estimating a homeowner's property and income taxes, and preparing an amortization schedule. We provide students with actual materials drawn from several community sources to give them a real-life context for course work.

KEYWORDS: Student motivation, house buying, simple interest amortized loan, amortization schedule, mathematics of finance, finite mathematics, active learning, group project, practical applications.

INTRODUCTION

Providing motivation is always a central concern in teaching a mathematics course. This is especially true in lower level or terminal courses such as college algebra, liberal arts mathematics, or finite mathematics where students may lack background, motivation, or even interest in the subject. In this paper, we offer a description of our experiences with a house-buying project that has enabled us for many years to convince students in lower level courses of the importance of mathematics. Students acquire valuable

skills in the context of financing a new home and, having seen the relevance of mathematics in their everyday lives, are more receptive to learning new material.

In the project discussed below, students consider many aspects of house buying, including choosing a fixed-rate mortgage program subject to financial constraints, calculating property tax and income tax savings, preparing an amortization schedule, estimating prepaid finance charges, and computing an unpaid balance. Once students are made aware of the financial considerations of making a dream home a reality, it is a short step to convincing them that it is worth studying the mathematics involved in buying a house.

PROJECT OVERVIEW

Surely everyone has some corner of his or her heart reserved for a dream home. It may be no more than a vague notion of someday having a "place of one's own". It may be a fully imagined three-bedroom ranch, with even the carpets and countertops already selected. College students in particular are at a stage of life when they are beginning to think about making such a dream into a reality. However, when it becomes a reality, this dream home will likely cost thousands, hundreds of thousands, of dollars. For a $120,000 home, a borrower will pay more than $360,000 over the life of a standard thirty-year mortgage at 9.5%. In addition to the mortgage, a prospective homebuyer must also take into account taxes, mortgage insurance, points, fees, and other expenses. As they become aware of the practical considerations of purchasing a home, students become very interested in the mathematics involved. Many of our lower level students excel in other fields, but unfortunately sometimes have the attitude that their mathematics course is just an irritating distribution requirement, with no bearing on what really interests them. For many, this house-buying project is the first time they have appreciated the impact of mathematics on their personal futures. In particular, adult learners, who may have already experienced the house-buying process themselves, are especially eager to learn the mathematics of home finance.

Although house-buying exercises are occasionally available at the high school level, fully developed activities such as we describe here are not as prevalent at the college level. This is unfortunate, since this material seems more appropriate for college and continuing education students than for younger learners. In addition, textbook problems, no matter how involved, are just that to most students—textbook problems. They can lack credi-

bility to students who assume that they are simply exercises contrived to provide practice using abstract formulas.

We designed this house-buying project to give our students hands-on experience with some of the calculations and analysis involved in buying a home. Students are given (hypothetically!) a fixed amount of money that must cover all components of the closing costs, and a monthly budget that must cover the mortgage payments, mortgage insurance, and property tax escrow. Within these constraints, they must select a fixed-rate home mortgage from among those offered by a local financial institution.

To engage our students, we want them to have genuine current local data for the mortgage programs, tax rates, fair market value, and mortgage insurance. However, the activity seems to proceed more smoothly if students do not have to assemble this data themselves. We have found that when students try to collect their own information, they may encounter resistance from bank or town officials, who can be somewhat impatient with a young person who clearly is not a customer or taxpayer. Thus, we collect information for the activity from area banks, real estate guides, and town offices to make the project as realistic as possible and we present it to our students in as raw a form as possible.

We do emphasize to our students that we are *not* real estate professionals nor tax advisors. There are many variations in the process of buying a house, such as adjustable rate mortgages, construction loans, and assumable mortgages, for example. We do not discuss home finance in such depth in our courses. We recommend to our students that they consult the many books available, talk to friends and family members, take a community house-buying course, and educate themselves thoroughly before committing to such a major purchase. We do hope that this project will give them the experience to approach buying a house with the confidence that they will be able to learn to understand the financing ultimately involved.

STUDENT ACTIVITY

To begin with, students receive handouts or go to a web page to access project information. They are given an advertisement with the sale price, picture, and brief description of their "dream home", as well as a bank sheet with the current fixed rate simple interest amortized loan programs they are to consider. The objective of the project is to decide which program is affordable (given certain financial constraints outlined below) and least expensive. There are also some additional components that may be used if time permits.

We typically use loan programs from a local financial institution for 10, 20, and 30 year loans with 0, 1, 2, and 3 points each (or some subset thereof). The points are a percentage of the loan amount that must be paid to the lending institution when the loan is taken out. For example, borrowing \$2,000 with a 2 point loan means that .02 $\times$ \$2,000 or \$40 must be paid initially, *in addition* to the usual interest paid over the term of the loan.

Each program has an interest rate, Annual Percentage Rate (APR), and maximum Loan to Value (LTV). In our courses, we discuss computing the APR and using it to compare loan programs with equal principals. For this project however, the APR is used only to determine prepaid finance charges (more on this to follow). The maximum LTV of the loans we typically consider is 95%, meaning that a maximum of 95% of the bank's appraised value of the house can be borrowed. We make the somewhat unrealistic simplifying assumption that the advertised price, sale price, and bank appraisal of the house are all equal. Students then need to allocate enough resources to cover a down payment of at least 5% of the sale price of the house.

Students are given an initial amount of savings and a limited monthly budget to cover the mortgage payments, mortgage insurance, and property tax escrow, discussed below. A portion of their savings will cover initial charges, such as prorated expenses, legal costs, inspection and appraisal fees, utility hookups, mortgage insurance prepayment, and other costs. The remainder will pay for the cost of points and down payment on their loan.

Some of the equations needed for the various objectives of the project are described below. We derive these equations in our courses, and they are also developed in many standard texts. We list the equations here to provide a sense of the mathematics that needs to be developed in a lower-level mathematics course so that this project would be appropriate and could run successfully.

- Down payment:

$$A = p(C - D) + D,$$

  where A is the amount of money available to cover the cost of points and down payment, p is the points in decimal form (for example, for 3 points, p would be 0.03), C is the cost of the house, and D is the down payment. In this project, students begin with a fixed amount of savings A to cover both the down payment and cost of points. The cost of points is computed on the amount borrowed, namely $C - D$. This means that the down payment can be found by solving for D in the equation given above. Through this, our students learn the important

relationship that the more money they allocate to the down payment, the less money they have to borrow, and the less interest they will have to pay over the term of the loan. Thus, for each loan option, they will want to make D as large as possible (with a minimum of 5% of the cost of the house) to decrease the interest they will owe.

- Simple interest amortized loan payments:

$$P(1+i)^n = pymt\left(\frac{(1+i)^n - 1}{i}\right),$$

where P is the principal, i is the monthly periodic interest rate, n is the total number of periods, and *pymt* is the amount of the monthly principal and interest payments.

- Estimating prepaid finance charges:

$$(P - fee)(1+i)^n = pymt\left(\frac{(1+i)^n - 1}{i}\right),$$

where P is the principal, i is the APR divided by 12, n is the total number of payment periods, and *pymt* is the amount of the monthly principal and interest payments. In this equation, solving for $fees$ gives an estimate of the prepaid finance charges. This equation is nearly the same as the previous one, except that the interest rate increases to the APR and the principal decreases to the legal loan amount, namely $P - fees$. The difference between the original principal and the legal loan amount yields an estimate of finance charges. Based on these calculations, students can determine if the financial institution charged any fees in their loan in addition to the cost of points.

- Unpaid balance:

$$Balance = P(1+i)^m = pymt\left(\frac{(1+i)^m - 1}{i}\right),$$

where P is the principal, i is the monthly periodic rate, m is the number of periods from the beginning of the loan to the present, and *pymt* is the amount of the monthly principal and interest payments. This equation results from taking the difference between the left and right sides of the simple interest amortized loan equation after m payment periods. The unpaid balance is the amount that the borrower would need to pay off the loan before it comes to term. In one of

the final parts of the project, students can determine how much they would still owe if they decide to pay off the loan before the end of the mortgage. This is useful when considering selling or refinancing the house, for example.

Armed with these equations, students determine the cost of each loan program that fits their financial constraints, and then select the one that has the least total cost. Specifically, the total cost of a loan is the total interest plus the cost of any points. For some loans, students may not be able to afford both the points and the minimum down payment. For others, the total monthly payment may be too much.

Other components of this project include becoming familiar with mortgage insurance, computing property taxes and property tax escrow, preparing an amortization schedule using a spreadsheet or software package, and estimating tax savings from mortgage interest. Often first-time home buyers have LTV ratios greater than 80%, and in this case, many lending institutions require them to carry mortgage insurance. Because of this, we provide our students with a monthly cost of mortgage insurance that they must include in their total monthly payments.

We also give our students the property tax rate and percent of fair market value for local property appraisal, so that they can estimate the monthly property tax escrow payments. To compute the taxable value of the property, students multiply the sale price of the house by the percent of fair market value on houses in the town. The tax rate is usually given as a dollar amount, for example \$1.25, which means that \$1.25 in taxes is charged for every \$100 of property value owned. Thus, the tax rate can be thought of as a percent—1.25%, or 0.0125. Then, to estimate the yearly taxes, they multiply the taxable value of the property by the tax rate. Dividing the result by 12 yields the amount of monthly property tax escrow that must be paid. This money is paid into an escrow account, a non-interest bearing account from which the bank pays the property taxes.

After comparing loan programs and choosing the least expensive affordable program, students prepare an amortization schedule for the first and last 12 months of the loan. Using this amortization schedule and assuming they are in a specific tax bracket, they can compute their income tax savings for the first year of their loan. To do this, they add up the interest portions of each of the first 12 monthly payments. Since this interest often can be deducted from a borrower's adjusted gross income when computing income tax, then the tax savings would be a percentage (determined by the tax bracket) of this total interest paid.

IMPLEMENTING THE PROJECT

This project requires only basic mathematical skills and some elementary concepts from financial mathematics, such as determining amortized loan payments. Because of this, it can be implemented in a variety of courses, including pre-calculus, finite mathematics, liberal arts mathematics, mathematics education, college algebra, and elementary business. To give maximum flexibility in course structure, the project is designed so that students can complete it outside of class. However, portions of it are short enough to be used as in-class activities.

The computations used to analyze the different aspects of house buying are substantial, but appropriate to any course requiring basic algebra skills. The financial equations for this project can be integrated into a course in different ways, depending on the instructional philosophy of the course. For example, the necessary equations may be developed in preceding classes, with the house-buying project given as a culminating practical application of the course material. Alternatively, the different aspects of financing a home can be used to motivate the development of new equations, as the need for each equation arises. There may even be courses where it is appropriate to simply present the formulas for students to use in the project without deriving them at all.

It is easy to modify the project for any geographic location or price range. A house/condominium/apartment can be found in a local real estate guide, current mortgage rates can be found at any bank, and tax information can be obtained through a telephone call to the appropriate town office. If a spreadsheet is used to compute the solutions, these regional modifications, as well as yearly updates, are easy to implement since only the raw data needs to be entered into the pertinent cells to generate answers for the whole project.

The full version of this house-buying project can readily be shortened if this would better serve a specific course. Although the main activity of the project is choosing the most advantageous mortgage program within given financial constraints, we have also discussed several ancillary topics such as prepaid finance charges, unpaid balance, property tax, and income tax savings. While these are important considerations for a prospective home buyer, they are independent of determining the best mortgage program, and can be omitted if a simpler version of this project is desired.

In our own courses, most students simply use hand-held calculators to complete all parts of the project, except the amortization schedule, which they prepare using a software package. If a software package or spreadsheet is not available, students can create an abbreviated (3 to 6 month) amor-

tization schedule using a hand calculator. In fact, they often gain a better understanding of the mechanics of a mortgage (realizing why it is called a *simple interest* amortized loan, for example) by working out these details by hand. However, the tedium of computing a lengthy schedule with only a calculator quickly provides great motivation for learning to use a spreadsheet or other software. Thus, although this project can be done using only a calculator, it is especially suited for courses that incorporate some kind of spreadsheet program or other software.

EVALUATION

This is a substantial project that requires students to demonstrate thorough mastery of basic financial concepts and computations, proficiency with a calculator or software program, and analytic ability in a real-life context. Because students take this project seriously and put so much time and effort into it, it is especially important to grade it seriously as well. We offer our students individual feedback during the project. For example, we encourage them to check their answers after completing the computations for just one of the loan programs. This verifies that they understand the questions and are using the correct values in the proper formulas before finishing the remaining several programs. Also, this can prevent student frustration at making the same small error in every loan program and having to redo the entire project.

As with any project of this kind, grading can be time-consuming. There are some strategies that may minimize the amount of time spent grading. One possibility is to assign these projects to groups rather than individuals. Groups of two to four students seem to work the best. When there are more than four students in a group, our experience has been that one or more of the students do not participate fully. Group dynamics can be both positive and negative, and this should be considered when deciding whether to assign a project to individuals or to groups. The use of spreadsheets also helps streamline grading. For example, if a group makes an error early on (such as incorrectly computing the down payment amount), this error can be entered into a copy of the spreadsheet. The resulting changes in the spreadsheet solutions can then determine if the group did all subsequent computations correctly.

To help our students organize the information and to further streamline the grading, we require our students to use a copy of the Mortgage Programs Chart below to list results for each loan program. If any loan program is unusable, then they should give a reason, such as insufficient down payment

or too costly total monthly payment. Once they determine a program is too expensive, they need not finish the computations for that program.

After deciding which program is both affordable and least expensive, students then use the Final Information chart below to organize the information for their chosen program. This chart includes optional portions of the project.

Mortgage Programs Chart

Number of years	
Points	
Rate	
APR	
Number of months	
Down payment	
Amount of loan	
Cost of points	
Monthly principal and interest payment	
Total monthly payment	
Total interest	
Total cost of loan	

Final Information Chart

Cost of house	
Total cash on hand	
Amount needed for closing costs other than points	
Amount left for points and down payment	
5% minimum down payment	
Taxable value of your property	
Yearly property taxes	
Monthly escrow tax payments	
Best affordable program (number of years/points)	
Rate for your program	
APR for your program	
Estimated prepaid finance charges for your program	
Cost of points for your program	
Total cost of your loan	
Income tax savings over the first 12 months of your program	
Unpaid balance of your loan after 10 years	

BENEFITS

In addition to demonstrating the relevancy of mathematics, this house-buying project provides several other benefits. It reinforces basic mathematical concepts, such as understanding percentages and order of operations, manipulating exponents, solving equations, and simplifying fractions. It gives students the opportunity to develop calculator and computer skills. It offers students a format other than traditional tests to demonstrate proficiency. Working on a project such as this in groups provides a venue for valuable student interaction. Perhaps most importantly, this project helps prepare students for the future since it involves exactly the same type of data and analysis crucial to forthcoming life decisions.

Student response to this project in our courses has been very positive. Students are motivated to do a significant amount of mathematical analysis. More importantly, they feel that the experience is valuable and the mathematics meaningful. Most of our students find this project relevant, and they seem to work much more diligently on it than routine textbook exercises. Students' written evaluations at the ends of our courses testify to the effectiveness of this project with comments such as:

> "Amortization schedules ... bank loans: all are helpful for graduating seniors!"
>
> "This class helped us prepare for life's real challenges such as ... mortgages ...."
>
> "I learned many practical things from this class such as ... how to figure out a mortgage, and many other practical experiences. This math class seemed much more 'real' than any other."

As a final bonus, this project has been both fun and educational for us, too. We enjoy picking just the right house with our students each semester (mountain chalets for the fall semester, lakeside cottages in the spring!). Tracking the changing mortgage rates over the years and chatting with town clerks can be both interesting and entertaining. Furthermore, both of us have bought houses during the time we have been using this project and have benefited from the very material we develop in class. One of us even found several hundred dollars worth of bank error in the closing papers as a result. Motivating and beneficial for students, fun for teachers, and profitable too—what more could we want?

BIOGRAPHICAL SKETCHES

George Ashline received his BS from St. Lawrence University, his MS from the University of Notre Dame, and PhD from the University of Notre Dame in 1994 in value distribution theory. He has taught at St. Michael's since 1995. He is a participant in Project NExT, a program created for new or recent PhD's in the mathematical sciences who are interested in improving the teaching and learning of undergraduate mathematics.

Joanna Ellis-Monaghan received her BA from Bennington College, her MS from the University of Vermont, and her PhD from the University of North Carolina at Chapel Hill in 1995 in algebraic combinatorics. She has taught at Bennington College, at the University of Vermont, and at St. Michael's College since 1992. She is a great proponent of active learning and has developed materials, projects, and activities to augment a variety of courses.