6.2
eg #7

a) ∃ at least two balls of each color:

$$\left(x^2 + x^3 + \dots\right)^3$$

we want the coef of x^{10} in here

$$\left(x^2\right)^3\left(1 + x + x^2 + \dots\right)^3 \quad \text{use \#2}$$

$$x^6 \cdot \left(\frac{1}{1-x}\right)^3 \quad \text{use \#5}$$

$$x^6\left(1 + \dots + \binom{r+3-1}{r}x^r + \dots\right)$$

to get x^{10}, need a $r = 4$ here

So coef of x^{10} is $\binom{4+3-1}{4} = \binom{6}{4}$

$$\binom{n}{r} = \frac{n!}{r!(n-r)!} \qquad \binom{6}{4} = \frac{6!}{4!\,2!}$$

${}_nC_r$

b. At most two red balls.

$$(1 + X + X^2)(1 + X + X^2 + \dots)^2$$

again want coef of X^{10}

$$(1 + X + X^2)\left(\frac{1}{1-X}\right)^2$$

$$(1 + X + X^2)\left(1 + \dots \binom{r+2-1}{r} X^r + \dots\right)$$

We want

$1 \cdot \text{coef } X^{10} + X \cdot \text{coef } X^9 + X^2 \cdot \text{coef } X^8$

$$\binom{10+2-1}{10} + \binom{9+2-1}{9} + \binom{8+2-1}{8}$$

ways.