$$\overbrace{(1+\underline{x})(1+\underline{x})\cdots(1+x)}^{n}$$

choose x from r of

these factors

to get an x^r

$$(1+x)^n = \sum_{r=0}^{n} \binom{n}{r} x^r$$

an example of a "generating function",
i.e. a function where the coefficients count something

warning

can be tautologies.

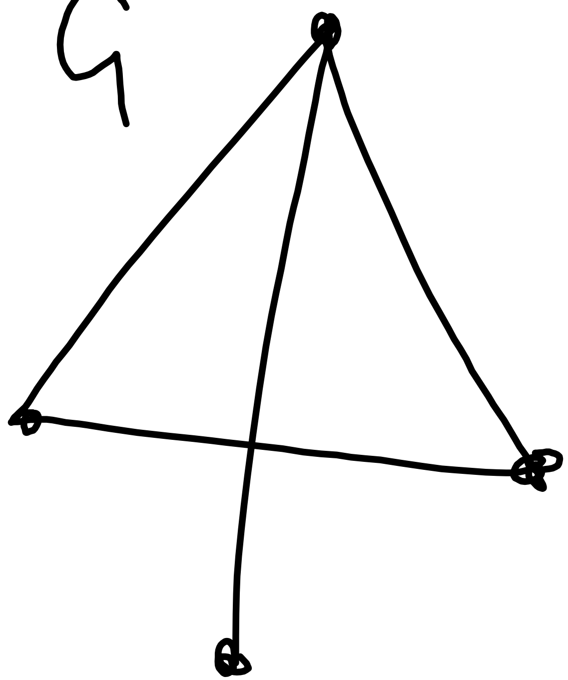
G a graph

eg $$f(G) = \sum_d n_d x^d$$

where n_d = # of vertices of degree n.

e.g.

G



$$f(G) = x + 2x^2 + x^3$$

$$\underbrace{(\overset{x^0 + x^1}{1+x})(1+x)\cdots\cdots(1+x)}_{n \text{ things}}$$

at most one of each thing

for a total of r things

Suppose we have

A, B, C

(red blocks) (Blue) (green)

Pick

up to 3 things of type A

$$\left(x^0 + x^1 + x^2 + x^3\right)$$

2 or 3 things of type B

$$\left(x^2 + x^3\right)$$

Any number of things of type C.

$$\left(x^0 + x^1 + x^2 + \cdots\right)$$

How many ways can we pick 11 things from Types A, B, C if we can have up to 3 of type A, 2 or 3 of type B and any number of type C?

Solution — the coefficient of

x^{11} in

$$(1+x+x^2+x^3)(x^2+x^3)(1+x+x^2+\dots)$$

examples

find all integer sols. to

$e_1 + e_2 + e_3 = r$ if $0 \le e_i \le 4$

$(1 + x + x^2 + x^3 + x^4)^3$

~~a~~nd take coef of r in this expanded.

eg. find all int. sols. to

$$e_1 + e_2 + e_3 + e_4 = r$$

if $0 \le e_i$

e_2, e_4 are odd

$e_4 \le 3$

$$(1 + x + x^2 + \dots)^2 \cdot (x + x^3 + x^5 \dots) \cdot (x + x^3)$$

eg distribute r identical objects into 7 different boxes with at least one object per box.

$$(x + x^2 + \cdots)^7$$

eg 3 boxes w/ at most 5 in the first box.

$$\left(1+x+x^2 \cdots x^5\right)\left(1+x+x^2 \cdots\right)^2$$