\[
\left(1 + \frac{x}{r}\right)\left(1 + \frac{x}{r}\right) \cdots \left(1 + \frac{x}{r}\right)
\]

choose \(x\) from one of these factors to get an \(X^r\)
\[(1 + x)^n = \sum_{r=0}^{n} \binom{n}{r} x^r\]

an example of a "generating function", i.e. a function where the coefficients count something
Warning can be tautologies.

e.g. a graph

\[ f(G) = \sum_{\mathbb{P}} n_d^d \]

where \( n_d \) = # of vertices of degree \( d \).
eq

\[ f(4) = x + 2x^2 + x^3 \]
\( x^0 + x^1 \)

\[(1 + x)(1 + x) \ldots (1 + x)\]

\( n \) things

at most one of each thing

for a total of \( r \) things
suppose we have

A, B, C
(red blocks) (Blue) (green)

pick
up to 3 things of type A
\((x^0 + x^1 + x^2 + x^3)\)

2 or 3 things of type B
\((x^2 + x^3)\)

any number of things of type C
\((x^0 + x^1 + x^2 + \ldots)\)
How many ways can we pick 11 things from Types A, B, C if we can have up to 3 of type A, 2 or 3 of type B, and any number of type C?
Solution - the coefficient of $x^n$ in

\[
(1 + x + x^2 + x^3)(x^2 + x^3)(1 + x + x^2 + \ldots)
\]
Examples

find all integer sols. to

\[ e_1 + e_2 + e_3 = r \quad \text{if} \quad 0 \leq e_i \leq 4 \]

\[ (1 + x + x^2 + x^3 + x^4)^3 \]

and take coeff of \( r \) in this expanded.
Example: find all integral solutions to

\[ e_1 + e_2 + e_3 + e_4 = r \]

\[ (1 + x + x^2 + \ldots) \cdot (1 + x^2 + x^4 + \ldots) \cdot (1 + x^3 + x^6 + \ldots) \cdot (1 + x^3) \]

if \( 0 \leq e_i \)

\( e_2, e_4 \) are odd

\( e_4 \leq 3 \)
Example: distribute $r$ identical objects into $7$ different boxes with at least one object per box.

$$(x + x^2 + \cdots)^7$$
\[ e^{x} \leq 3 \text{ boxes w/ at most } 5 \text{ in the first box.} \]

\[
\left( 1 + x + x^2 \ldots + x^5 \right) \left( 1 + x + x^2 \ldots \right)
\]