16.6

I. A parametric surface is a vector valued function

\[ \vec{r}(u,v) = \langle x(u,v), y(u,v), z(u,v) \rangle \]
II The tangent plane to \( \vec{r}(u, v) \)

has normal \( \vec{r}_u \times \vec{r}_v \)

so for example, the tangent plane
to \( \vec{r}(u, v) = \langle u^2, v^2, u-v \rangle \) at

\( (1,4, -3) \)  

so \( u = \frac{5}{2}, \ v = \frac{1}{2} \)

but \( h \cdot v = -\frac{3}{2} \) so

\( h = -1, \ v = \frac{3}{2} \)
has normal using

\[ \overrightarrow{r_u} = \langle 2u, 0, 1 \rangle \rightarrow \langle -2, 0, 1 \rangle \]

\[ \overrightarrow{r_v} = \langle 0, 2v, -1 \rangle \rightarrow \langle 0, 4, -1 \rangle \]

So \( n = \begin{vmatrix} i & j & k \\ -2 & 0 & 1 \\ 0 & 4 & -1 \end{vmatrix} = \langle -4, -2, -8 \rangle \)

or \( \langle 2, 1, 4 \rangle \) reduced
so the equation of the tangent plane
through \((1, 4, -3)\) \perp \((2, 1, 4)\)

is
\[
2(x - 1) + 1(y - 4) + 4(z + 3) = 0
\]
III surface area:

Surface area of \( \tilde{r}(u,v) \) for \( u, v \) in some region \( D \)

is \( \int_{\tilde{r}_u \times \tilde{r}_v} \, dA \)