Suppose $C$ is a curve

$$\vec{r}(t) = \langle x(t), y(t) \rangle$$

and $f(x,y)$ is a function whose domain includes $C$.

Then the area of the "curtain" $f(x,y)$ above $C$ is

$$\int_C f(x,y) \, ds$$

where $s$ is the arclength function for $C$.
recall that

\[ S_a = \int \sqrt{\left( \frac{dx}{dt} \right)^2 + \left( \frac{dy}{dt} \right)^2} \, dt \]

so

\[ \frac{ds}{dt} = \sqrt{\left( \frac{dx}{dt} \right)^2 + \left( \frac{dy}{dt} \right)^2} = \left| r'(t) \right| \]

\[ ds = \left| r'(t) \right| \, dt \]
also "f(x, y) over C"

really means \( f(x(t), y(t)) \)

\[ a \leq t \leq b \]
So
\[ \int_{a}^{b} f(t) \, dt = \int_{a}^{b} f(x(t), y(t)) \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \, dt \]

Use to understand
Use to compute
Use to remember what to do