eg. Find the global max/min of

\[ f(x, y) = 2x^2 + x + y^2 - 2 \]

on \[ S = \{ (x, y) \mid x^2 + y^2 = 4 \} \]

Find 1st derivatives and set equal to zero:

\[ f_x = 4x + 1 \quad 4x + 1 = 0 \Rightarrow x = -\frac{1}{4} \]

\[ f_y = 2y \quad 2y = 0 \Rightarrow y = 0 \]

Only critical point is \((-\frac{1}{4}, 0)\)

Now do 2nd deriv stuff:

\[ f_{xx} = 4 \quad \text{so} \quad D = 4 \cdot 2 - 6^2 = 8 > 0 \]

\[ f_{yy} = 2 \quad f_{xx} \left(-\frac{1}{4}, 0\right) = 4 > 0 \]

\[ f_{xy} = 0 \]

So \[ f \left(-\frac{1}{4}, 0\right) = -2 \frac{1}{8} \] is a local min.
Now check the boundary

The boundary of $S$ is $x^2 + y^2 = 4$

so $y^2 = 4 - x^2$

$y = \pm \sqrt{4-x^2}$ with $-2 \leq x \leq 2$.

so we are looking for max/min of

$f(x, \pm \sqrt{4-x^2}) = 2x^2 + x + 4 - x^2 - 2 = x^2 + x + 2$

ie find max/min of $g(x) = x^2 + x + 2$ on $[-2, 2]$. 