1. Let $\vec{u}$ be any unit vector $\langle a, b \rangle$.

and let $\langle x_0, y_0, z_0 \rangle$ be a point on the surface.

Then $D_{\vec{u}} f(x_0, y_0) = f_x(x_0, y_0) \cdot a + f_y(x_0, y_0) \cdot b$

is the directional derivative of $f(x,y)$ in the direction $\vec{u} = \langle a, b \rangle$. 
\[ D_u f = \langle f_x, f_y \rangle \cdot \langle a, b \rangle \]

\[ \langle f_x, f_y \rangle \text{ gets used (a lot!)} \]

Notation: \[ \nabla f = \langle f_x, f_y \rangle \]
The max value of $D_u f(x, y)$ at the point $(x, y)$ is $|\nabla f|$ and occurs when $
abla f$ has the same direction as $\nabla f$. 
Proof

\[ \nabla f = \nabla f \cdot \hat{u} \]

\[ = |\nabla f| \cdot |\hat{u}| \cdot \cos \theta \]

\[ = |\nabla f| \cdot \cos \theta \]

This is biggest when

\[ \cos \theta = 1 \iff \theta = 0 \iff \hat{u} \text{ is parallel to } \nabla f. \]
\[ z^2 = (x-1)^2 + \frac{(y-2)^2}{4} \]

Write everything on left and call it \( F(x, y, z) \)

Tangent plane at \((a, b, c)\)

\[ \nabla F \cdot (\langle x, y, z \rangle - \langle a, b, c \rangle) = 0 \]

or \[ F_x(x-a) + F_y(y-b) + F_z(z-c) = 0 \]

Note that the normal to this plane

is \( \nabla F \)

Symmetric equations for the normal line are

\[ \frac{x-a}{F_x} = \frac{y-b}{F_y} = \frac{z-c}{F_z} \]
\( \nabla \cdot (u \nabla v) = u \nabla v \cdot \nabla + u \cdot \nabla v \)
\[ \nabla u v = \langle (u v)_x, (u v)_y \rangle \]
\[ = \langle u_x v + u v_x, u_y v + u v_y \rangle \]

and
\[ u \nabla u = u \langle u_x, u_y \rangle = \langle u v_x, u v_y \rangle \]
\[ v \nabla u = v \langle u_x, u_y \rangle = \langle v u_x, v u_y \rangle \]

add to get \[ \langle u v_x + u v_x, u v_y + v u_y \rangle \]