\[ m = \int_{c} p(x, y) \, ds \]

\[ m = \int_{-\pi/2}^{\pi/2} k \, dt = \pi k \]

\[ r(t) = (\cos t, \sin t) \]

\[ -\frac{\pi}{2} \leq t \leq \frac{\pi}{2} \]

\[ ds = \sqrt{(-\sin t)^2 + (\cos t)^2} \, dt \]

\[ = 1 \, dt \]
\[
\overline{x} = \frac{1}{m} \int_{C} x k \, ds = \frac{1}{k \pi} \int_{0}^{\pi/2} x k \cdot 1 \, dt
\]
16.3

\int \mathbf{F} \cdot d\mathbf{r} = f(r(b)) - f(r(a))

Same answer!
\[ \int_{a}^{b} \mathbf{F} \cdot d\mathbf{r} = \int_{a}^{b} f_x(x(t), y(t)) \mathbf{x}'(t) + f_y(x(t), y(t)) \mathbf{y}'(t) \, dt \]

\[ = \int_{a}^{b} \frac{df}{dx} \frac{dx}{dt} + \frac{df}{dy} \frac{dy}{dt} \, dt \]

by chain rule

\[ = \left. f(x(t), y(t)) \right|_{a}^{b} \]

\[ f(x(b), y(b)) - f(x(a), y(a)) \]

\[ = f(\mathbf{r}(b)) - f(\mathbf{r}(a)) \]
if \[ \vec{F}(x, y) = \langle P(x, y), Q(x, y) \rangle \]
on an open, simply connected
region \( D \), and \( P, Q \) have
continuous \( P_x, P_y, Q_x, Q_y \) on \( D \)
and
\[
\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}
\]
Then \( \vec{F} \) is "conservative"

i.e. \( \vec{F} = \nabla f \) for
some \( f \)
There is no specific formula for finding $f$. So that $\hat{F} = \nabla f$ but there is a process that usually works.
\[ F(x, y) = (x^2 + y)^2 i + (y^2 + x)^2 j \]

\[
\frac{dp}{dy} = 1, \quad \frac{dq}{dx} = 1, \quad \text{so yep, conservative}
\]
we want $f$ so that
\[ \vec{F}(x,y) = \nabla f(x,y) = \langle f_x(x,y), f_y(x,y) \rangle \]
so we want
\[ f_x(x,y) = x^2+y \]
so $f(x,y) = \int x^2+y \, dx$
\[ = \frac{1}{3} x^3 + y \times + g(y) \]
so \( f(x, y) = \int x^2 + y \, dx \)

\[ = \frac{1}{3} x^3 + y \cdot x + g(y) \]

Now consider \( f_y(x, y) \)

from here

\( f_y = x + g'(y) \)

But from \( F(x, y) = <x^3 + y, y^2 + x> \)

we have that \( f_y = y^2 + x \)

Thus \( y^2 + x = x + g'(y) \)

\[ y^2 = g'(y) \]

so \( \int y^2 \, dy = g(y) \)

\[ \frac{1}{3} y^3 + C = g(y) \]
Put this all together to get

\[ f(x,y) = \frac{1}{3} x^3 + y x + \frac{1}{3} y^3 + C \]