\[
\text{eg } \int \int \frac{x + y}{y} \, dy \, dx
\]

\[
\int_{x^2}^{x^{-1}} y^{-1} + x^{-1} \, dy
\]

\[
\left| x \ln(y) + x^{-1} y^2 \right|_{1}^{2}
\]
\[\iint_D f(x, y) \, dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x, y) \, dy \, dx\]
\[ \iint_D f(x,y) \, dA = \int_c^{h_1(y)} \left( \int_{h_2(y)}^{p(y)} f(x,y) \, dx \right) \, dy \]
Really yucky $g_2(x)$

eq here do in 3 pieces

$$\int_A \int_B f(x,y) \, dx \, dy$$

$$\int_C \int_D f(x,y) \, dx \, dy$$

$$\int_A \int_B f(x,y) \, dy \, dx$$

etc.
4. \[ \iint_D A = \text{area of } D \]

5. if \( m \leq f(x, y) \leq M \)

Then \( m \cdot \text{area of } D \leq \iint_D f(x, y) \, dA \leq M \cdot \text{area of } D \)
\[ \int \int_{D} x \sin y \, dA \quad D = \left\{ (x,y) \mid 0 \leq y \leq \frac{\pi}{2}, \quad 0 \leq x \leq \cos y \right\} \]

\[ \int_{0}^{\frac{\pi}{2}} \int_{0}^{\cos y} x \sin y \, dx \, dy \]

First do \[ \int_{0}^{\cos y} x \sin y \, dx \]

\[ = \frac{x^2}{2} \sin y \bigg|_{0}^{\cos y} \]

\[ = \frac{\cos^2 y - 0}{2} \sin y \]

Next do \[ \int_{0}^{\frac{\pi}{2}} \]

\[ = \frac{1}{2} \left( -\frac{\cos^3 y}{3} \right) \bigg|_{0}^{\frac{\pi}{2}} \]

\[ = -\frac{1}{6} \left( 0 - 1 \right) = \frac{1}{6} \]

\[ \frac{1}{2} \left( \frac{1}{2} \right)^3 - \frac{1}{2} \left( \frac{1}{3} \right)^3 \]

\[ = \frac{1}{2} \left( \frac{1}{3} - \frac{1}{27} \right) = \frac{1}{2} \left( \frac{1}{3} \right) = \frac{1}{6} \]
\( z = 3x^2 + y^2 \) above the region bounded by \( y = x \) and \( x = y^2 - y \).

\[
\begin{align*}
\frac{\partial}{\partial y} y^2 - y &= 2y - 1 \\
0 &= y^2 - 2y \\
0 &= y(y - 2) \\
y &= 0, 2
\end{align*}
\]

\[
\iiint_R 3x^2 + y^2 \, dx \, dy = \frac{104}{35}
\]
\[ \int_0^{\pi/2} \int_0^{\sin x} f(x, y) \, dy \, dx \]

Diagram:
- \( y = \sin x \)
- \( x = \sin y \)

Shaded area:
- \( 0 \leq x \leq \pi/2 \)
- \( 0 \leq y \leq \sin x \)
\[
\int_0^1 \frac{x}{x^2 + 1} \, dx 
\]

Let \( x = \sqrt{y} \), then \( x^2 = y \) and \( dx = \frac{1}{2\sqrt{y}} \, dy \). The integral becomes:

\[
\int_0^1 \frac{\sqrt{y}}{y + 1} \cdot \frac{1}{2\sqrt{y}} \, dy = \frac{1}{2} \int_0^1 \frac{1}{\sqrt{y} + 1} \, dy
\]

First, evaluate the integral:

\[
\frac{1}{2} \left[ \sqrt{x^3 + 1} \right]_0^1 = \frac{1}{2} \left( \sqrt{2^3 + 1} - \sqrt{0^3 + 1} \right) = \frac{1}{2} \sqrt{9} = \frac{3}{2}
\]

Next, evaluate the inner integral:

\[
\int_0^1 \frac{1}{\sqrt{x^3 + 1}} \, dx = \frac{1}{2} \int_0^1 \frac{2}{\sqrt{u^3 + 1}} \, du
\]

Let \( u = x^3 + 1 \), then \( du = 3x^2 \, dx \) and \( \frac{1}{3} \, du = x^2 \, dx \).

Now, the integral becomes:

\[
\frac{1}{3} \int_{u(0)}^{u(1)} \frac{1}{\sqrt{u}} \, du = \frac{1}{3} \int_{1}^{2} \frac{1}{\sqrt{u}} \, du
\]

Evaluate the integral:

\[
\frac{1}{3} \left[ \frac{2}{\sqrt{u}} \right]_1^2 = \frac{2}{3} \left( \frac{2}{\sqrt{2}} - \frac{2}{\sqrt{1}} \right) = \frac{2}{3} \left( \frac{2}{\sqrt{2}} - 2 \right)
\]

The final result is:

\[
\int_0^1 \frac{x}{x^2 + 1} \, dx = \frac{2}{9} \left( \frac{\sqrt{2}}{2} - 1 \right)
\]