\[ (xy)^{1/2} = 1 + x^2 y \]

\[ \frac{dy}{dx} = -\frac{x}{F_y} \]

\[ x^{1/2} y^{1/2} - 1 - x^2 y = 0 \]

\[ F_x = \frac{F}{2} x^{-1/2} y^{1/2} - 0 - 2xy \]
Directional Derivative

Let \( \hat{u} \) be any unit vector in the \( x-y \) plane.

The slope of the tangent line to \( f(x,y) \) in the \( \hat{u} \) direction at \( (x_0, y_0) \) is

\[ D_{\hat{u}} f(x_0, y_0) \]
\[ \nabla_{\mathbf{u}} (f(x, y)) = f_x(x, y) \mathbf{a} + f_y(x, y) \cdot \mathbf{b}. \]

\[ = \langle f_x, f_y \rangle \cdot \langle a, b \rangle \]

\[ = \nabla \varphi \cdot \mathbf{u}. \]
The max value of $D_u f(x,y)$

is $|D f(x,y)|$ and occurs when $\nabla f$ has the same direction

as $D f$
Lesson:

\[ D_u f = Df \cdot \hat{u} \]

\[ = |Df| |\hat{u}| \cos \theta \]

Max when \( \cos \theta = 1 \)

\[ \text{when } \cos \theta = 1 \]

\[ \text{get max value } D_u f = |Df| \]

\[ \text{i.e. } Df \text{ and } \hat{u} \text{ are } \perp \]
e.g. $f(x, y, z) = x + \frac{y}{z}$ at $(4, 3, -1)$

Find max rate of change and the direction in which it occurs.

$$Df = \left< 1, \frac{1}{z}, -\frac{y}{z^2} \right>$$ plug in $(4, 3, -1)$

get

$$Df = \left< 1, -1, -3 \right>$$

So max rate of change $|Df| = |<1, -1, -3>| = \sqrt{11}$.

in the direction

$$\vec{u} = \frac{1}{\sqrt{11}} \left< 1, -1, -3 \right>.$$