\[ x^2 y^3 = \cos (x + y + z) \]

Find \( \frac{dz}{dx} \)

**Bookkeeping**

- \( x \) is the variable
- \( z \) is a function of \( x \)
- \( y \) is constant
here.

\[ y^2 + xy \frac{dz}{dx} = -\sin(x+y+z) \cdot \left(1+\frac{dz}{dx}\right) \]

\[ xy \frac{dz}{dx} + \sin(x+y+z) \frac{dz}{dx} = -y \frac{dz}{dx} - \sin(x+y+z) \]

\[ \frac{dz}{dx} = \frac{-y \frac{dz}{dx} - \sin(x+y+z)}{xy + \sin(x+y+z)} \]
\[ f(x, y, z) = xe^y + ye^z + ze^x \]

\[ f_{xxz} \]

\text{means}

\[ \left( \frac{\partial^3 f}{\partial z \partial x \partial x} \right) = \frac{\partial^3 f}{\partial z \partial x \partial x} = \frac{2}{\partial^2} \left( \frac{\partial}{\partial x} \left( \frac{\partial f}{\partial x} \right) \right). \]
\( f(x) \)

1st do
\[
f_x = e^y + ze^x
\]
\[
f_{xx} = ze^x
\]
\[
f_{xxx} = e^x
\]
Clairaut's theorem.

If \( f(x,y) \) is defined on a disk \( D \) containing \( (a,b) \) and both \( f_{xy} \) and \( f_{yx} \) are continuous on \( D \), then \( f_{xy}(a,b) = f_{yx}(a,b) \).
\( f(x, y) \) a function. \((a, b, f(a, b))\) a point on the surface.

The line tangent to \( f(x, b) \)

has equation:

\[
z - f(a, b) = \left. \frac{d}{dx} f(x, b) \right|_{x=a} \cdot (x - a)
\]

\[= f_x(a, b) \]

\[\frac{z - f(a, b)}{x - a}, \quad y - y_1 = m(x - x_1), \quad \text{slope of tangent line.}\]
The slope is
\[
f_x(a, b)
\]
so has direction vector
\[
\langle 1, 0, f_x(a, b) \rangle
\]
(in x-direction).

Similarly
\[
\langle 0, 1, f_y(a, b) \rangle
\]
in y direction.
Take cross product to get the tangent plane at \((a, b, f(a, b))\).

\[
\langle 0, 1, f_y \rangle \times \langle 1, 0, f_x \rangle = \langle f_x, f_y, -1 \rangle
\]

So equation of tangent plane is

\[
f_x(a, b)(x-a) + f_y(a, b)(y-b) - (z - f(a, b)) = 0
\]

Boudi:

\[
f_x(x_0, y_0)(x-x_0) + f_y(x_0, y_0)(y-y_0) = z-z_0
\]