Beautiful short cut

\[ \int_{C} \mathbf{Df} \cdot d\mathbf{r} = f(\mathbf{r}(b)) - f(\mathbf{r}(a)) \]

where \( C \) is \( \mathbf{r}(t) \) for \( a \leq t \leq b \)
\[ \mathbf{F} = \langle p, q \rangle \]

with \[ \frac{dp}{dy} = \frac{dq}{dx}, \] then \[ \mathbf{F} \] is conservative

\[ \mathbf{F} = \nabla f \] for some \( f \).
Bad news—

No easy formula for finding $f$, given $\overrightarrow{F} = \nabla f$

but there is a (long-ish) process that usually works.
Find \( \int_C \vec{F} \cdot dr \) if \( C = \vec{r}(t) = \langle te^t, (t^2 + 1)e^t \rangle \) for \( 0 \leq t \leq 1 \).

\( \vec{F}(x, y) = e^x z + (1 + 2x)xe^y \)

Check if \( \vec{F} \) is conservative.

\[
\frac{\partial P}{\partial y} = 2ye^y, \quad \frac{\partial Q}{\partial x} = 2e^y \quad \text{equal?} \quad \text{yay!}
\]
\[ \mathbf{F} = \nabla f(x,y) \text{ where} \]
\[ f(x,y) = xe^{2y} + y \]

Thus
\[ \oint_C \mathbf{F} \cdot d\mathbf{r} = f(\mathbf{r}(1)) - f(\mathbf{r}(0)) \]
\[ = f(1e^{1}, 2) - f(0,1) = e \cdot e^2 + 2 - 1 \]
\[ = e^5 + 1 \]