**Example:** find the abs. max/min of

\[ f(x,y) = 2x^2 + x + y^2 - 2 \] on

\[ S = \{(x,y) \mid x^2 + y^2 \leq 4\} \]

\[ f_x = 4x + 1 \quad \Rightarrow \quad 4x + 1 = 0 \quad \Rightarrow \quad x = -\tfrac{1}{4} \]

\[ f_y = 2y \quad \Rightarrow \quad 2y = 0 \quad \Rightarrow \quad y = 0 \]

*critical point is \((\tfrac{1}{4}, 0)\)*
\[ f_{xx} = 4 \]
\[ f_{yy} = 2 \]
\[ f_{xy} = 0 \]

so \( D = 4 \cdot 2 - 0^2 = 8 > 0 \)

and \( f_{xx} \left( -\frac{1}{4}, 0 \right) = 4 > 0 \)

Thus \( f \left( -\frac{1}{4}, 0 \right) = 2 \cdot \frac{1}{16} - \frac{1}{4} + 0 - 2 = -2\frac{1}{8} \)

is a relative minimum.
The boundary of $S$ is $x^2 + y^2 = 4$

So $y^2 = 4 - x^2$

$y = \pm \sqrt{4 - x^2}$

with $-2 \leq x \leq 2$

\[
\phi(x, \pm \sqrt{4 - x^2}) = 2x^2 + x + 4 - x^2
\]

\[
= x^2 + x + 2
\]

So now find the (calc I) abs max/min of $x^2 + x + 2$ on $[-2, 2]$
Now \( x^2 + x + 2 \) has a min when 
\[ 2x + 1 = 0 \quad \text{or} \quad x = -\frac{1}{2} \]

So... rel min on boundary occurs
when \( x = -\frac{1}{2}, \ y = \pm \sqrt{4 - \frac{1}{4}} = \pm \sqrt{\frac{15}{4}} \)
and \( f(-\frac{1}{2}, \pm \sqrt{\frac{15}{4}}) = \frac{7}{4} \)
Also need to check the end points:
\[ f(-1, \pm \sqrt{4-4}) = 4 \]
\[ f(1, \pm \sqrt{4-4}) = 8 \]

Thus \( -\frac{1}{2} \) is smallest and \( 8 \) is biggest
\( f(x,y) \) has \( \text{abs min of} \ -\frac{1}{2} \text{ at} \ (-\frac{1}{4}, 0) \)
and \( \text{abs max of} \ 8 \text{ at} \ (2,0) \).