Tangent vector in $x$-direction is $\langle 1, 0, f_x \rangle$

in $y$-direction is $\langle 0, 1, f_y \rangle$
The normal to the tangent plane is the cross product
\[<0, 1, f_y> \times <1, 0, f_x> \]
\[= <f_x, f_y, -1> \]

and the tangent plane goes through some point \((a, b, f(a, b))\)
So eq. of tangent plane is

\[ f_x(a,b) (x-a) + f_y(a,b) (y-b) - 1 (z-f(a,b)) = 0 \]

Book: point is \((x_0, y_0, z_0)\)

get

\[ f_x(x_0,y_0) (x-x_0) + f_y(x_0,y_0) (y-y_0) = z-z_0 \]

Calc 1: line \( f(x_0) (x-x_0) = y-y_0 \)
\[ \Delta z = f(x+\Delta x, y+\Delta y) - f(x, y) \]

(increment in \( z \))

\[ \frac{dz}{dx} = f_x(x_0, y_0)(x-x_0) + f_y(x_0, y_0)(y-y_0) \]

\[ Z - Z_0 = f_x(x_0, y_0)(x-x_0) + f_y(x_0, y_0)(y-y_0) \]
If we write \( \Delta x = x - x_0 \) \( \Delta y = y - y_0 \),

we get

\[
\Delta z = f_x(xy) \Delta x + f_y(xy) \Delta y
\]

**Note:** \( \Delta z \approx \Delta z \)

Analogously in higher dimension
Example: Tangent plane to \( z = e^{x \ln y} \) at \((3, 1, 0)\):

\[
f_x = e^{x \ln y} \\
f_y = \frac{e^{x}}{y}
\]

\[
f_x(3, 1) = 0 \\
f_y(3, 1) = e^3
\]

Tangent plane is:

\[
0(x-3) + e^3(y-1) = z - 0
\]

or

\[
e^3y - e^3 = z
\]
Example 16

Since \( \Delta z \approx dz \)
\[
dz \approx f(x + \Delta x, y + \Delta y) - f(x, y)
\]
so \( dz + f(x, y) \approx f(x + \Delta x, y + \Delta y) \)

Here \( f(x, y) = \ln(x - 3y) \)

And we want \( f(6, 9, 2.06) \)

So we let \( x = 7, \Delta x = -0.1 \)
\[
y = 2, \quad \Delta y = 0.6
\]
\[
f_x = \frac{1}{x - 3y}, \quad f_x(7, 2) = 1
\]
\[
f_y = \frac{-3}{x - 3y}, \quad f_y(7, 2) = -3
\]
Thus

\[ dz = 1(-.1) + -3(.66) = -.28 \]

and \( f(7, 2) = \ln(7-6) = \ln 1 = 0 \)

Thus

\[ dz + f(7, 2) \approx f(6.9, 2.06) \]

\[-.28 \approx f(6.9, 2.06) \]