\[ \frac{dy}{dx} = f(x) \]

**eg.**
\[ \frac{dy}{dx} = x^3 + 3x + 2 \]

\[ y = \int (x^3 + 3x + 2) \, dx \]
Numerical approach to diff eq.

By looking at direction field

e.g. \( y' = 1 - xy \)
Eulers method on \( y' = 1 - xy \)
given \( y(0) = 0 \), estimate \( y(1) \) using step size of 0.2.

\[ y(0) = 0 = y_0, \quad x_0 = 0 \]

Slope at \( (0,0) \) is \( y' = 1 - 0 \cdot 0 = 1 \)

i.e. \( y'(x_0, y_0) = 1 \)

1st tangent line is

\[ y = m(x - x_0) + y_0 = 1 \cdot (x - 0) + 0 = x \]

Now move 0.2 along this line to get the next \( y \) value: so the next point on our estimated curve is \((0.2, 2)\)

\[ (x_1, y_1) = (0.2, 2) \]
Now do it again...  

at (1.2, 2) new slope is  

\[ y'(1.2, 2) = 1 - .2 \times 2 = 1 - .4 = .96 \]

new point is  

\[ y_2 = y'(1.2, 2)(x_2 - x_1) + y_1 \]

\[ y_2 = .96 \times .2 + .2 \]
The general formula is

\[ y_n = y'(x_{n-1}, y_{n-1}) \cdot \text{step} + y_{n-1} \]

where \( \text{step} \) is given

and \( x_n = x_0 + n \cdot \text{step} \)