\[ y = 4x^2 \quad 2x + y = 6 \]

\[ x = \frac{-\sqrt{2}}{2} \]

Rotate about \( x \)-axis

Find this
\[ 4x^2 = 6 - 2x \]
get \( x = 1, -\frac{3}{2} \)

Set up
\[
\int_{0}^{\frac{\pi}{2}} \pi y \left( \frac{\sqrt{4y} - \frac{-\sqrt{4y}}{2} \right) \, dy
\]

\[
\int_{\frac{\pi}{2}}^{0} \pi y \left( \frac{\sqrt{4y} - \frac{-\sqrt{4y}}{2} \right) \, dy
\]
\[ W = F \cdot d \] if force is constant.

but if force changes continuously with distance, as \( f(x) \) where \( x \) is distance

then can estimate work by

\[ W \approx \sum f(x_i) \Delta x \] a Riemann Sum

so \( W = \int_a^b f(x) \, dx \)
Hooke's law says

\[ f(x) = kx \]

here given

\[ f(0.1) = 25 \]

so

\[ k \cdot 0.1 = 25 \]

\[ k = 250 \]

Thus

\[ f(x) = 250x \]

\[ w = \int_{0}^{0.05} 250x \, dx = 0.315 \, J \]
At time $t$, $x = 2t$ so \( \frac{x}{2} \)

Bucket weighs $4 + 40 - 2t$

$= 44 - 1x$

So work done at $x_c^*$

Thus $W \approx \sum_{i=1}^{80} (44 - 1x_i^*) \Delta x$

So $W = \int_0^{84} 44 - 1x \, dx \approx 3200$
\[ \int_{a}^{b} f(x) \, dx = f_{\text{avg}}(x) \cdot (b-a) \]

A.K.A. Mean Value Theorem for integrals.
defined as
\[ f_{\text{avg}} = \frac{1}{b-a} \int_{a}^{b} f(x) \, dx \]

e.g. #10.5) find \( f_{\text{avg}} \) of \( \sqrt{x} \) on \([0,4]\)
\[ f_{\text{avg}} = \frac{1}{(4-0)} \int_{0}^{4} x^{\frac{1}{2}} \, dx \]

5) to find \( c \), solve
\[ f(c) = f_{\text{avg}} \text{ for } c \]