A function \( f(x) \) is one-to-one if \( f(x_1) = f(x_2) \implies x_1 = x_2 \). This is equivalent to \( f(x) \) passing the horizontal line test.

One to one functions have inverses. So if \( f \) is one-to-one with domain \( A \) and range \( B \), then it has an inverse \( f^{-1} \) with domain \( B \) and range \( A \) so that

\[
 f(a) = b \iff f^{-1}(b) = a \\
\text{i.e. } f(f^{-1}(x)) = x = f^{-1}(f(x))
\]

Steps for finding \( f^{-1}(x) \) given \( f(x) \):

1. \( y = f(x) \)
2. Switch \( x \) and \( y \)
3. Solve for \( y \)
4. The answer is \( f^{-1}(x) \)

Recall: the inverse \( a^x \) is \( \log_a x \). So \( \log_a x = y \iff a^y = x \) and \( a^x = x = \log_a x \)

\[
\text{if } \frac{\ln x}{\ln a} = \frac{\ln a^x}{\ln a} \iff \ln a^x = \ln x
\]

\[
\ln a^x = \ln x \\
\ln \left( \frac{a^x}{x} \right) = \ln \frac{a^x}{x} \\
\ln a^x - \ln x = \ln \frac{a^x}{x} \\
\ln a^x = \ln \left( \frac{a^x}{x} \right)
\]