A function $f(x)$ is one-to-one

if $f(x_1)=f(x_2) \Rightarrow x_1 = x_2$ ("implies")

This equivalent to $f(x)$ passing the horizontal line test.

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One to one functions have inverses

so if f is one-to-one with domain A and range B, then it has an invers f^{-1} with domain B and range A so that

$$f(a)=b \iff f^{-1}(b)=a$$

(ie $f(f^{-1}(x)) = x = f^{-1}(f(x))$)

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Steps for finding $f^{-1}(x)$ given $f(x)$

write

1. $y = f(x)$
2. switch x and y
3. solve for y
4. the answer is $f^{-1}(x)$

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Recall: the inverse of a^x is $\log_a x$

so $\log_a x = y \iff a^y = x$

and $a^{\log_a x} = x = \log_a a^x$

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eg. $f(x)=\frac{1+e^x}{1-e^x}$. find $f^{-1}(x)$

1. $y = \frac{1+e^x}{1-e^x}$
2. $x = \frac{1+e^y}{1-e^y}$
3. solve for y

$x(1-e^y) = 1+e^y$

$x - xe^y = 1+e^y$

$x-1 = e^y + xe^y$

$x-1 = (1+x)e^y$

$\frac{x-1}{1+x} = e^y$

$\ln\left(\frac{x-1}{1+x}\right) = \ln e^y$

$\ln\left(\frac{x-1}{1+x}\right) = y$

4. $f^{-1}(x) = \ln\left(\frac{x-1}{1+x}\right)$

$= \ln(1+x) - \ln(1-x)$

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