

A function  $f(x)$  is one-to-one if  $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$  <sup>implies</sup>

This equivalent to  $f(x)$  passing the horizontal line test.

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One to one functions have inverses

So if  $f$  is one-to-one with domain  $A$  and range  $B$ , then it has an inverse  $f^{-1}$  with domain  $B$  and range  $A$  so that

$$f(a) = b \Leftrightarrow f^{-1}(b) = a$$

(ie  $f(f^{-1}(x)) = x = f^{-1}(f(x))$ )

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Steps for finding  $f^{-1}(x)$  given  $f(x)$ .

write

1.  $y = f(x)$
2. switch  $x$  and  $y$
3. solve for  $y$ .
4. the answer is  $f^{-1}(x)$ .

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Recall: the inverse of  $a^x$  is  $\log_a x$ .

so  $\log_a x = y \Leftrightarrow a^y = x$

and  $a^{\log_a x} = x = \log_a a^x$

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ex.  $f(x) = \frac{1+e^x}{1-e^x}$ . find  $f^{-1}(x)$

1.  $y = \frac{1+e^x}{1-e^x}$
2.  $x = \frac{1+e^y}{1-e^y}$
3. solve for  $y$ 

$$x(1-e^y) = 1+e^y$$

$$x - x e^y = 1 + e^y$$

$$x - 1 = e^y + x e^y$$

$$x - 1 = (1+x) e^y$$

$$\frac{x-1}{1+x} = e^y$$

$$\ln\left(\frac{x-1}{1+x}\right) = \ln e^y$$

$$\ln\left(\frac{x-1}{1+x}\right) = y$$
4.  $f^{-1}(x) = \ln\left(\frac{x-1}{1+x}\right)$   

$$= -\ln(1+x) - \ln(1-x)$$

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