Theorem: If \( f'(x) \) exists at \( a \), then \( f(x) \) is continuous at \( a \).

Proof:
We need to show that \( \lim_{x \to a} f(x) = f(a) \)

\( (\text{def} \text{ of continuous}) \)

Given that \( f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} \)

\( (\text{def} \text{ of } f(x) \text{ exists at } a) \)

\[
\lim_{x \to a} f(x) - f(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} \cdot (x - a)
\]

So \( \lim_{x \to a} (f(x) - f(a)) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} \cdot \lim_{x \to a} (x - a) \)

\[
\lim_{x \to a} f(x) - f(a) = \lim_{x \to a} \frac{f(a)}{x - a} \cdot (x - a)
\]

\[
\lim_{x \to a} (f(x) - f(a)) = f'(a) \cdot 0
\]

So \( \lim_{x \to a} f(x) = f(a) \)