**Theorem:** If \( f(x) \) and \( g(x) \) are continuous at \( x=a \), then so are \( f \pm g, \; pg, \; cg, \; c^0f \), and \( f/g \) (where \( c \) is any constant except when \( g(x) = 0 \)).

Cor: all polynomials are continuous everywhere.

Cor: all rational functions \( (f(x))/g(x) \) are continuous everywhere except where \( g(x) = 0 \).

So \( g(x) = \frac{x^2-9x+5}{(x-1)(x+2)} \)

is continuous except at \( x=1, \; 2 \).
**The Intermediate Value Theorem**

If $f(x)$ is continuous on $[a, b]$ and $N$ is between $f(a)$ and $f(b)$, then $\exists \ c \in [a, b]$ such that $f(c) = N$.

Let's consider $\frac{1}{\sqrt{x}}$ on $(0, 1)$.

1. \[ \sqrt{x} = 1-x \text{ on } (0, 1) \]
2. \[ x - 1 + \sqrt{x} = 0 \]
3. There exists $x$ in $(0, 1)$ such that $f(x) = x - 1 + \sqrt{x}$ is zero.

Note:
- $f(0) = -1$
- $f(1) = 0$

and 0 is between -1 and 1.

and $f(x)$ is continuous on $[0, 1]$.

Thus, for $f(x)$ is continuous on $[0, 1]$, $\exists \ c \in [0, 1]$ such that $f(c) = 0$. 