\[
\lim_{x \to 2} \frac{3x^2 - 6}{x - 2} = 10
\]

\[f(x) = \frac{3x^2 - 6}{x - 2}\]
\[L = 10\]
\[a = 2\]
\[|x - 2| < \delta\]
\[|f(x) - 10| < \epsilon\]

Back to continuity...

One-sided continuity:

\[f(x)\]

Here \( f(x) \) is continuous at \( a \) from the right, i.e.

\[\lim_{x \to a^+} f(x) = f(a)\]

Recall: \( f(x) \) is continuous at \( a \) if

\[\lim_{x \to a} f(x) = f(a)\]

Now \( f(x) \) is continuous on an interval \( I \) if \( f(x) \) is continuous at every point in \( I \).

Theorem:

If \( f(x) \) and \( g(x) \) are continuous at \( a \), then:

- \( f(x) = g(x) \)
- \( f(x)g(x) \)
- \( f(x)/g(x) \)

and \( f(x)/g(x) \) when \( g(a) \neq 0 \) are all continuous at \( a \) as well.

Consider:

Polynomials are continuous everywhere.

Rational functions are continuous everywhere except where denominator is zero.

\( g(x) = \frac{x^5 - 3x + 11}{x - 2} \) is cont. everywhere except \( x = 2 \)

\( g(x) = \frac{x^5 - 3x + 11}{x - 2} \) is cont. everywhere except \( x = 2 \).
Example:

\[ \lim_{x \to a} g(x) = b, \text{ then } \lim_{x \to a} f(g(x)) = f(b) \]

\[ \lim_{x \to a} g(x) \]

\[ \lim_{x \to a} f(g(x)) = f(\lim_{x \to a} g(x)) \]

Theorem:

Intermediate Value Theorem:

If \( f(x) \) is continuous on \([a, b]\) and \( N \) is between \( f(a) \) and \( f(b) \), then there exists \( c \) in \([a, b]\) such that \( f(c) = N \).

**Picture**

- Plot of \( f(x) \)
- Point \( N \)
- \( f(a) \) and \( f(b) \)
- Line showing \( f(c) = N \)

Example:

\[ \sqrt{x} = 1 - x \]

Let \( f(x) = \sqrt{x} - 1 \)

\[ f(1) = 0 \]

So, by the intermediate value theorem, there is some \( c \) in \((0, 1)\) such that \( f(c) = 0 \)

\[ \sqrt{c} - 1 = 0 \]

\[ c = 1 \]

So, \( f(c) = 0 \) for \( c = 1 \)