defⁿ
A function is one-to-one if

"implies"

$$f(x_1)=f(x_2) \Rightarrow x_1=x_2$$

This is equivalent to the "horizontal line test": f(x) is one-to-one if every horizontal line intersects the graph at most once.

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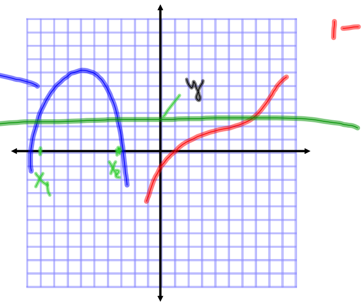
f(x) not 1-1



1-1

A horizontal line

note $f(x_1)=y$
$f(x_2)=y$
but $x_1 \neq x_2$.

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if f(x) is 1-1 with domain A and range B, then it has an inverse, $f^{-1}(x)$ with domain B and range A such that

"if and only if"

$$f(a)=b \Leftrightarrow f^{-1}(b)=a$$

$$\left(\text{and } f(f^{-1}(x))=x=f^{-1}(f(x))\right)$$

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The graphs are reflections about the line $y=x$.

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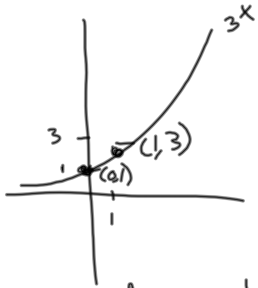

The inverse of a^x is $\log_a x$

where $\log_a x=y \Leftrightarrow a^y=x$

e.g. $\log_3 9=2$ since $3^2=9$

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Laws of Logs

$\log(xy)=\log x+\log y$

$\log\left(\frac{x}{y}\right)=\log x-\log y$

$\log x^r = r\log x$

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Change of base:

$$\log_a x = \frac{\ln x}{\ln a}$$

Proof: let $y = \log_a x$, so $a^y = x$ (defn)

so $\ln a^y = \ln x$ (take ln of both sides)

$y \ln a = \ln x$ (law of logs)

$y = \frac{\ln x}{\ln a}$ (divide)

$\therefore \log_a x = \frac{\ln x}{\ln a}$ (since $y = \log_a x$)

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eg. find f^{-1} if $f(x) = \frac{1+e^x}{1-e^x}$

$x = \frac{1+e^y}{1-e^y}$

$x(1-e^y) = 1+e^y$

$x - xe^y = 1+e^y$

$x-1 = e^y + xe^y$

$x-1 = (1+x)e^y$

$\frac{x-1}{x+1} = e^y$

$\ln\left(\frac{x-1}{x+1}\right) = \ln e^y$

$\ln\left(\frac{x-1}{x+1}\right) = y$

Thus $f^{-1}(x) = \ln\left(\frac{x-1}{x+1}\right)$

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