Calc 11-19-08.notebook

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\[ y' = 15x + c = 0 \text{ has no real roots in } [2, 3] \]

If there were two roots, say \( x = a \) and \( x = b \), then \( f(a) = 0 \), \( f(b) = 0 \).

So the MVT says there exists \( c \) such that

\[ f'(c) = \frac{f(b) - f(a)}{b - a} = \frac{0 - 0}{b - a} = 0 \]

So there are two roots between 2 and 3.

\[ 2^2 \]

For \( 1 \leq x \leq 4 \), there exists \( c \) in \((1, 4)\) such that

\[ f'(c) = \frac{f(4) - f(1)}{4 - 1} = 2 \]

So \( f(1) - 10 \geq 2 \)

\[ f(4) = 16 \]

So one of the roots is 16.

\[ \frac{4}{2} \]

**Def:**

\( f(x) \) is increasing on an interval \( I \) if for all \( a < b \) in \( I \), \( f(a) \leq f(b) \).

\( f(x) \) is decreasing on an interval \( I \) if for all \( a < b \) in \( I \), \( f(b) \leq f(a) \).

Then:

\[ f'(x) > 0 \text{ on } I \Rightarrow f(x) \text{ is increasing} \]

\[ f'(x) < 0 \text{ on } I \Rightarrow f(x) \text{ is decreasing} \]

Proof: Suppose \( f(x) > 0 \) and \( a < b \) in \( I \).

Then MVT says there exists \( c \) in \((a, b)\) so

\[ f'(c) = \frac{f(b) - f(a)}{b - a} \]

\[ f'(c) > 0, \quad \text{so} \quad f(b) > f(a) \]

**First derivative test:**

If \( c \) is a critical number of \( f(x) \),

\[ f'(c) \text{ DNE or } = 0. \]

Then

\[ f(x) \text{ inc.} \quad \text{at} \quad c \text{ inc.} \quad \Rightarrow f(c) \text{ is a local max}. \]

\[ f(x) \text{ dec.} \quad \text{at} \quad c \text{ dec.} \quad \Rightarrow f(c) \text{ is a local min.} \]

**Second derivative:**

\[ f''(x) > 0 \Rightarrow f(x) \text{ is concave up} \]

\[ f''(x) < 0 \Rightarrow f(x) \text{ is concave down} \]
Second derivative test:

If \( f''(c) = 0 \) and

\[
\begin{align*}
  f''(c) &< 0, \quad \text{then } f(c) \text{ is a local max} \\
  f''(c) &> 0, \quad \text{then } f(c) \text{ is a local min}
\end{align*}
\]

C is an inflection point if

\[ f'' < 0 \text{ for } x < c \quad \text{and} \quad f'' > 0 \text{ for } x > c \]

\[ f'(x) = 12x^2 + 4x - 6 \]

Set to zero and solve:

\[ 12x^2 + 4x - 6 = 0 \]

\[ 2x^2 + x - 1 = 0 \]

Use the quadratic formula to find the roots:

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

Use these to test the intervals for concavity.