4.1 #67

\[ f(x) = \sqrt{x-x^2} \]

\[ f(x) = x \cdot \left( x-x^2 \right)^{\frac{1}{2}} \]

\[ f'(x) = 1 \cdot \left( x-x^2 \right)^{\frac{1}{2}} + x \cdot \frac{1}{2} \left( x-x^2 \right)^{-\frac{1}{2}} \cdot (1-2x) \]

\[ f'(x) \text{ is undefined when } x - x^2 = 0 \]
\[ x \cdot (1-x) = 0 \]
\[ x = 0 \text{ or } x = 1 \]
Also set \( f'(x) = 0 \) and solve for \( x \)

\[
\frac{(x-x^2)^{1/2} + x \cdot \frac{1}{2} (x-x^2)^{1/2} (1-2x)}{(x-x^2)^{-1/2}} \cdot \frac{1}{x \cdot \frac{1}{2} (1-2x)} = 0
\]

\[
\frac{x-x^2 + \frac{x}{2} - x^2}{\sqrt{x-x^2}} = 0
\]

\[
\frac{\frac{3}{2} x - 2 x^2}{\sqrt{x-x^2}} = 0
\]

\[
\frac{3}{2} x - 2 x^2 = 0
\]

\[
x \left( \frac{3}{2} - 2x \right) = 0
\]

\[
so \ x = 0, \ or \ \frac{3}{2} - 2x = 0
\]

\[
\frac{3}{2} = 2x
\]

\[
\frac{3}{4} = x
\]

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<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
</tr>
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<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>3/4</td>
<td>.324</td>
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</tbody>
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Since $f(x)$ is only defined on $[0, 1]$, we have:

- Absolute min = 0
- Absolute max = .324

Also, end points...
\[ f(x) = x^{4/5} (x-4)^2 \]
\[ f'(x) = \frac{4}{5} x^{-1/5} (x-4)^2 + x^{4/5} 2(x-4)^1 \]

At \( x = 0 \),

\[ f'(x) \text{ does not exist} \]

Set \( f'(x) = 0 \)

\[ \frac{4}{5} x^{-1/5} (x-4)^2 + x^{4/5} 2(x-4)^1 = 0 \]

\[ x^{-1/5} (x-4) \left( \frac{4}{5} (x-4) + x^{1/2} \right) = 0 \]

\[ (x-4) \left( \frac{4}{5} x - \frac{16}{5} \right) = 0 \]

\[ x^{1/5} \]

\[ 2 \frac{(x-4)(7x-8)}{5 x^{1/5}} = 0 \]

So set \( (x-4)(7x-8) = 0 \)

\[ x = 4, \quad \frac{8}{7} \]
<table>
<thead>
<tr>
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<tbody>
<tr>
<td>0</td>
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<tr>
<td>4</td>
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<tr>
<td>$\frac{8}{7}$</td>
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</table>
\[ w^2 + s^2 = D^2 \]

\[ \frac{ds}{dt} = 60 \]

\[ \frac{dw}{dt} = 25 \]

\[ \frac{dD}{dt} \]

\[ 2w \cdot \frac{dw}{dt} + 2s \cdot \frac{ds}{dt} = 2D \frac{dD}{dt} \]

when \( t = 2 \), \( w = 50 \), \( s = 120 \)

\[ D = \sqrt{50^2 + 120^2} = 130 \]

\[ 50 \cdot 25 + 120 \cdot 60 = 130 \cdot \frac{dD}{dt} \]

\[ 6S = \frac{dD}{dt} \]
Proof that if \( f(x) \) has a local (relative) extremum at \( c \) then either \( f'(c) \) DNE or \( f'(c) = 0 \).

(Fermat's thm — you do min in class in chll probs)

we'll do max in class
We need to start with a more careful defn of local max.

\[ f(c) \] is a local max if there is some \( \delta \) such that 
\[ \forall x \in \mathbb{R}, |x-c| < \delta \implies f(x) \leq f(c) \].

i.e. if \( |h| < \delta \) then
\[ f(c) \geq f(c+h) \]
\[ f(c) \leq f(c+h) \]
Now if \( f(c) \) is a local max and \( f'(c) \) DNE, we're done.

So assume \( f'(c) \) does exist. The we need to show that \( f'(c) = 0 \).
Well, recall

\[ f'(c) = \lim_{h \to 0} \frac{f(c+h) - f(c)}{h} \]

Since \( h \to 0 \), we can assume \( |h| < \delta \), so that \( f(c+h) - f(c) \) is neg.
So if $0 < h < \delta$

\[
\lim_{h \to 0^+} \frac{f(c+h)-f(c)}{h} \quad \text{pos.}
\]

but if $-\delta < h < 0$

\[
\lim_{h \to 0^-} \frac{f(c+h)-f(c)}{h} \quad \text{neg.}
\]
but $f'(c)$ exist, so
\[ \lim_{x \to c, \text{ from left}} f(x) = \lim_{x \to c, \text{ from right}} f(x), \]
so
\[ \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = 0 \]
\[ \therefore f'(c) = 0 \quad \checkmark \]