3.8

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\[ y = \ln(x^3 - 7) \]

\[ f(x) \text{ at } (2, 0) \]

Find tangent line

\[ f' \text{ at } (2, f(2)) \]

\[ y - f(a) = m(x-a) \]

\[ y - y_1 = m(x-x_1) \]

\[ y = f'(a)(x-a) + f(a) \]

\[ y = f'(2)(x-2) + 0 \]

\[ \frac{dy}{dx} = \frac{3x^2}{x^3 - 7} \]

So \[ f'(2) = \frac{12}{1} = 12 \]

So the equation of the tangent line is

\[ y = 12(x-2) + 0 \]

\[ y = 12x - 24 \]
Find \( f^{(n)}(x) \) if \( f(x) = \ln(x-1) \)

\[
\begin{align*}
f'(x) &= \frac{1}{x-1} = (x-1)^{-1} \\
f''(x) &= -(x-1)^{-2} \\
f'''(x) &= -(-2)(x-1)^{-3} = 2(x-1)^{-3} \\
f^{(4)}(x) &= 2(-3)(x-1)^{-4} = -2\cdot3(x-1)^{-4} \\
f^{(5)}(x) &= +2\cdot3\cdot4(x-1)^{-5} \\
\ldots \\
f^{(n)}(x) &= (-1)^{n+1} \cdot (n-1)! \cdot (x-1)^{-n}
\end{align*}
\]
eg I have a rectangle with area 10. The width is decreasing at a rate of 0.5 per sec. How fast is the height changing when the width is 2?
1. The rate we know is 
\[ \frac{dw}{dt} = -0.5 \]

2. The rate we want to know is 
\[ \frac{dh}{dt} \]

3. An equation relating \( w \) and \( h \) is 
\[ wh = 10 \]

4. Differentiate both sides with respect to \( t \)

So 
\[ \frac{dw}{dt} \cdot h + w \cdot \frac{dh}{dt} = 0 \]

5. Now plug stuff in: we want \( \frac{dh}{dt} \)
when \( w = 2 \), we know \( wh = 10 \)
so when \( w = 2 \), get \( 2h = 10 \), or \( h = 5 \)
we get \(-0.5 \cdot 5 + 2 \cdot \frac{dh}{dt} = 0 \)

6. Solve for \( \frac{dh}{dt} \)
\[ -2.5 + 2 \cdot \frac{dh}{dt} = 0 \]
\[ \frac{dh}{dt} = \frac{2.5}{2} = 1.25 \]