\[ f(x) = x^{4/3}(x-4)^2 \]

\[ f'(x) = \frac{4}{3}x^{-1/3}(x-4)^2 + x^{4/3} \cdot 2(x-4) \]
\[ = x^{-1/3}(x-4) \left( \frac{4}{3}(x-4) + x \cdot 2 \right) \]
\[ = x^{-1/3}(x-4) \left( \frac{4}{3}x - \frac{16}{3} + 2x \right) \]
\[ = x^{2/3}(x-4) \left( \frac{14}{3}x - \frac{16}{3} \right) \]
\[ = \frac{2x}{3}x^{-1/3}(x-4) \left( 7x - 8 \right) \]

Critical points: \( x = 0 \) (\( f'(0) \) undefined)
\( x = 4 \) \( f'(4) = 0 \)
\( x = \frac{8}{7} \) \( f'(8/7) = 0 \)
4.2 Rolle's Theorem

If
1. \( f(x) \) is continuous on \([a, b]\)
2. \( f'(x) \) exists on \((a, b)\)
3. \( f(a) = f(b) \)

Then \( \exists \) \( c \in (a, b) \) s.t.

\[ f'(c) = 0 \]
Proof:

if \( f(x) = k, \text{ constant} \)

\[ \forall x \in [a, b] \]

then \( f'(x) = 0 \quad \forall x \in (a, b) \)

so any \( c \in (a, b) \) "works"

i.e. \( f'(c) = 0 \quad \forall c \in (a, b) \).
otherwise \( f \) has either a local max or a local min in \((a, b)\), say at \( c \).

By Fermat's theorem,

\[
f'(c) = 0, \quad \text{since } f(x) \text{ exists on } (a, b) \text{ (given)}.
\]
Mean Value Theorem

If 1. \( f(x) \) is continuous on \([a, b]\)

2. \( f'(x) \) exists on \((a, b)\)

Then

\[ \exists \ c \in (a, b) \text{ such that} \]

\[ f'(c) = \frac{f(b) - f(a)}{b - a} \]
slope is $f'(c)$

slope is $\frac{f(b) - f(a)}{b - a}$