Try \( \lim_{x \to 0} f(x) = 0 \)

if \( \epsilon = \frac{1}{2} \), cannot find a \( \delta \), so no limit.
\[ \lim_{x \to a} f(x) = \infty \iff \]

\[ \forall M > 0 \exists \delta \text{ s.t. } \]

\[ |x - a| < \delta \implies f(x) > M \]
Solving $f(x) = m$
\[
\frac{1}{(x+3)^4} = 100000
\]

\[
(x+3)^4 = \frac{1}{100000}
\]

\[
x + 3 = \frac{1}{10}
\]

\[
x = -3 + \frac{1}{10}
\]
2.5

If \( f \) is continuous at \( b \),

and \( \lim_{x \to a} g(x) = b \)

then \( \lim_{x \to a} f(g(x)) = f(b) \)

i.e. \( \lim_{x \to a} f(g(x)) = f(\lim_{x \to a} g(x)) \)
Theorem: Intermediate Value Theorem

Theorem:

If $f(x)$ is continuous on a closed interval $[a, b]$ and $N$ is between $f(a)$ and $f(b)$, then there exists $c \in (a, b)$ such that $f(c) = N$. 
Example 48

\[ \sqrt[3]{x} = 1 - x \quad \text{in (0, 1)} \]

ie a sol to

\[ 0 = 1 - x - \sqrt[3]{x} \]

So we take \( f(x) = 1 - x - \sqrt[3]{x} \)

and note that \( f(x) \) is cont on \([0, 1]\)

and \( f(0) = 1, \quad f(1) = -1 \)

\( 0 \) is between 1 and -1

So the intermediate value theorem says \( \exists \, c \in (0, 1) \) with

\[ f(c) = 0, \quad \text{ie } 1 - c - \sqrt[3]{c} = 0, \; \text{so c is a root.} \]