

eg.
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$$f(x) = \frac{x}{x+1}$$

(domain
 $x \neq -1$)

(warm up)

$$f(2) = \frac{2}{2+1} = \frac{2}{3}$$

$$f(2+h) = \frac{2+h}{2+h+1} = \frac{2+h}{3+h}$$

$$f(x+h) = \frac{x+h}{x+h+1}$$

$$\frac{f(x+h) - f(x)}{h} = \frac{\frac{x+h}{x+h+1} - \frac{x}{x+1}}{h}$$

$$\frac{\frac{(x+h)(x+1)}{(x+h+1)(x+1)} - \frac{x(x+h+1)}{(x+h+1)(x+1)}}{h}$$

$$\frac{(x+h)(x+1) - x(x+h+1)}{(x+h+1)(x+1)}$$

$$= \frac{x^2 + x + hx + h - (x^2 + xh + x)}{(x+h+1)(x+1)}$$

$$= \frac{h}{(x+h+1)(x+1)} = \frac{h}{(x+h+1)(x+1)} \cdot \frac{1}{h}$$
$$= \frac{1}{(x+h+1)(x+1)}$$

Another way

$$\frac{\frac{x+h}{(x+h+1)} - \frac{x}{x+1}}{h} = \frac{(x+h+1) \cdot (x+1)}{(x+h+1)(x+1)}$$

$$\frac{(x+h)(x+1) - x(x+h+1)}{h(x+h+1)(x+1)}$$

$$\frac{x^2 + x + hx + h - x^2 - xh - x}{h(x+h+1)(x+1)}$$

$$= \frac{h}{h(x+h+1)(x+1)} = \frac{1}{(x+h+1)(x+1)} \quad \checkmark$$

(general thoughts about domain/range as "shadows")

