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Exercise 8.3

Find all incongruent solutions to the following congruences.

a) $7x \equiv 3 \pmod{15}$

$\gcd(7, 15) = 1$ so there is only one solution.

Solution: $x = 9$

$$7(9) \equiv 3 \pmod{15}$$

$$63 \equiv 3 \pmod{15}$$

b) $6x \equiv 5 \pmod{15}$

No Solutions	$6(1) \equiv 6 \pmod{15}$	$6(9) \equiv 9 \pmod{15}$
	$6(2) \equiv 12 \pmod{15}$	$6(10) \equiv 0 \pmod{15}$
	$6(3) \equiv 3 \pmod{15}$	$6(11) \equiv 6 \pmod{15}$
	$6(4) \equiv 9 \pmod{15}$	$6(12) \equiv 12 \pmod{15}$
	$6(5) \equiv 0 \pmod{15}$	$6(13) \equiv 3 \pmod{15}$
	$6(6) \equiv 6 \pmod{15}$	$6(14) \equiv 9 \pmod{15}$
	$6(7) \equiv 12 \pmod{15}$	$6(15) \equiv 0 \pmod{15}$
	$6(8) \equiv 3 \pmod{15}$	

c) $x^2 \equiv 1 \pmod{8}$

Solutions:

$$x = 1, 3, 5, 7$$

$$1^2 \equiv 1 \pmod{8}$$

$$2^2 \equiv 4 \pmod{8}$$

$$3^2 \equiv 1 \pmod{8}$$

$$4^2 \equiv 0 \pmod{8}$$

$$5^2 \equiv 1 \pmod{8}$$

$$6^2 \equiv 4 \pmod{8}$$

$$7^2 \equiv 1 \pmod{8}$$

$$8^2 \equiv 0 \pmod{8}$$

d) $x^2 \equiv 2 \pmod{7}$

Solutions:

$x = 3, 4$

$1^2 \equiv 1 \pmod{7}$

$2^2 \equiv 4 \pmod{7}$

$3^2 \equiv 2 \pmod{7}$

$4^2 \equiv 2 \pmod{7}$

$5^2 \equiv 4 \pmod{7}$

$6^2 \equiv 1 \pmod{7}$

$7^2 \equiv 0 \pmod{7}$

e) $x^2 \equiv 3 \pmod{7}$

No Solutions

$1^2 \equiv 1 \pmod{7}$

$2^2 \equiv 4 \pmod{7}$

$3^2 \equiv 2 \pmod{7}$

$4^2 \equiv 2 \pmod{7}$

$5^2 \equiv 4 \pmod{7}$

$6^2 \equiv 1 \pmod{7}$

$7^2 \equiv 0 \pmod{7}$

Extension:

$x^3 \equiv 3 \pmod{7}$

No Solutions

$1^3 \equiv 1$

$2^3 \equiv 1$

$3^3 \equiv 6$

$4^3 \equiv 1$

$5^3 \equiv 6$

$6^3 \equiv 6$

$7^3 \equiv 0$