Section 8
Problem 8.2

Problem 8.2: Suppose that
\[ ac \equiv bc \pmod{m} \]
and also assume \( \gcd(c, m) = 1 \). Prove that \( a \equiv b \pmod{m} \).

Proof: Linear Congruence Theorem States:
When \( \gcd(a, m) = 1 \) and the congruence statement
is:
\[ ax \equiv c \pmod{m} \]
This has only one solution:
\[ x \equiv \frac{c}{a} \pmod{m} \]

Since we know \( \gcd(c, m) = 1 \), then:
\[ ac \equiv bc \pmod{m} \]
can be written as:
\[ a \equiv \frac{bc}{c} \pmod{m} \]
\[ \therefore \ a \equiv b \pmod{m} \]

Extension: Two Examples

(a) \( a = 47, c = 107, b = 35, m = 6 \)
\[
\begin{align*}
47 \equiv 35 \pmod{6} \\
47 = \frac{35 \cdot 107}{107} \pmod{6}
\end{align*}
\]
\[ 47 \equiv 35 \pmod{6} \]
\[ \therefore 6/47 - 35 \equiv 6/12 \checkmark \]

What is \( \frac{1}{107} \) \( \pmod{6} \)?
i.e. find \( x \) such that \( 0 \leq x < 6 \)
so that \( x \cdot 107 \equiv 1 \pmod{6} \)
\[ \Rightarrow x = 5 \Rightarrow 535 \equiv 1 \pmod{6} \]

(b) \( a = 26, c = 53, b = 16, m = 5 \)
\[
\begin{align*}
26 \equiv 16 \pmod{5} \\
26 = \frac{16 \cdot 53}{53} \pmod{5}
\end{align*}
\]
\[ 26 \equiv 16 \pmod{5} \]
\[ \therefore 5/26 = 16 \]
\[ \Rightarrow 5/10 \checkmark \]