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Problem 7.3

Give a proof by induction of each of the following formulas.

a.) $\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \ldots + \frac{1}{n(n-1)} = \frac{n-1}{n}$

$n=2 \quad \frac{2-1}{2} = \frac{1}{2}$ ✓

$n=3 \quad \frac{1}{2} + \frac{1}{6} = \frac{2}{3} = \frac{3-1}{3}$ ✓

$n=4 \quad \frac{1}{2} + \frac{1}{6} + \frac{1}{12} = \frac{3}{4} = \frac{4-1}{4}$ ✓

Assume true for $n \leq k$, $\frac{k-1}{k}$, ie $\sum_{n=2}^{k} \frac{1}{(n-1)n} = \frac{k-1}{k}$

Prove true for $n = k+1$, $\frac{(k+1)-1}{k+1} = \frac{k}{k+1}$

$$n=k+1 \implies \frac{k-1}{k} + \frac{1}{k(k+1)} = \frac{k-1}{k}\left(\frac{k+1}{k+1}\right) + \frac{1}{k(k+1)}$$

$$= \frac{k^2-1}{k(k+1)} + \frac{1}{k(k+1)}$$

$$= \frac{k^2 \cancel{-1} \cancel{+1}}{k(k+1)}$$

$$= \frac{k^2}{k(k+1)}$$

$$= \frac{k}{k+1}$$ ✓

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Problem 7.5

a) Find the first $\mathbb{M}$ - primes

$$\boxed{5, 9, 13, 17, 21, 29}$$

b) Find an $\mathbb{M}$ -number n that has two different factorizations as a product of $\mathbb{M}$ - primes

$n = 2205 \rightarrow k = 551$ st $n = 4(551) + 1 = 2205$

$$2205 = 5 \cdot 21 \cdot 21 \qquad 2205 = 5 \cdot 49 \cdot 9$$

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Extension:

Welcome to S-world, where the only numbers that exist leave a remainder of 1 when divided by 7. the only S-numbers that exist are of the form 7k+1

What are the first 10 S-numbers? What are the first 6 S-primes?

$$\left.\begin{aligned} 7(1)+1 &= 8\\ 7(2)+1 &= 15\\ 7(3)+1 &= 22\\ 7(4)+1 &= 28\\ 7(5)+1 &= 36\\ 7(6)+1 &= 43 \end{aligned}\right\} \text{First 6 S-primes!}$$

$$\begin{aligned} 7(7)+1 &= 50\\ 7(8)+1 &= 57\\ 7(9)+1 &= 64\\ 7(10)+1 &= 71 \end{aligned}$$