Group B
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Problem 7.3 (b) / 7.5

For problem 7.3 (b) we have to prove by induction that the formula $\frac{n(n+1)(n+2)}{6}$ is the formula for finding the sum of the series of n^2 for any *n*. So, we assumed true for $n \leq 4$.

$$N = 2$$
$$1^2 + 2^2 = 5$$
$$\frac{2(2+1)(4+1)}{6} = 5$$
$$N = 3$$
$$5 + 3^2 = 14$$
$$\frac{3(3+1)(6+1)}{6} = 14$$
$$N = 4$$
$$14 + 4^2 = 30$$
$$\frac{4(4+1)(8+1)}{6} = 30$$

So, this all checks out. Now that we know that up to n=4 works, we can check if it works for some (n-1) and n. So now, we can assume that

$$\sum_{n=1}^{N-1} n^2 = \frac{(N-1)N(2(N-1)+1}{6}$$ by induction hypothesis.

Thus,

$$\sum_{n=1}^{N-1} n^2 = \sum_{n=1}^{N-1} n^2 + N^2 = \frac{(N-1)N(2(N-1)+1}{6} + N^2$$

Now, by simplifying this out, we get the final answer of:

$$\frac{N(N+1)(2N+2)}{6}$$

For Problem 7.5, we are in the M-World. This means that the only numbers in existence are numbers congruent to $1 = m \bmod 4$, or $4t+1$ for any *t*.

a.) Find the first 6 M-Primes (not including 1).
5, 9, 13, 17, 21, 39

None of these can be divided by another M-number other than 1.

b.) Find an M-number that can be factorized by M-primes two different ways.

For this problem, we tried some trial and error, not knowing if there was any algebraic method of solving the problem. However, after many tries and lots of written out M-numbers, we finally found 441 to be an M-number that can be factorized into M-primes two different ways. First of all, we know that 441 is an M-number because $\frac{441}{4} = 110$ has a remainder of 1.

Now, since 441 is a square of 21, which is an M-prime, it can be factored into 21^2, which in M-world would be a factorization of two M-primes. The other way it can be factored is by 9*49 (both of which are M-primes).