Final Draft 7.3(a) and 7.4
Group A

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7.3(A) QUESTION:

Give proof by induction of the following formula:

$$1+2+3+\cdots+n=\frac{n(n+1)}{2}$$

So $T_n=\frac{n(n+1)}{2}$ and $T_n=T_{n-1}+n$

now, we assume true for $n-1$

$$T_{n-1}=\frac{(n-1)[(n-1)+1]}{2}$$

$$=\frac{n^2-n}{2}$$

So, $T_n=T_{n-1}+n$

$$\frac{n(n+1)}{2}=\frac{n^2-n}{2}+n$$

$$\frac{n(n+1)}{2}=\frac{n^2-n}{2}+\frac{2n}{2}$$

$$\frac{n(n+1)}{2}=\frac{n^2-n+2n}{2}$$

$$\frac{n(n+1)}{2}=\frac{n^2+n}{2}$$

$$\frac{n(n+1)}{2}=\frac{n(n+1)}{2} \quad \checkmark$$

7.4 a) Describe all E primes.

Answer: E primes are numbers that cannot be factored into two even numbers. Therefore, they are prime because you cannot use any odd numbers in the even world.

b) Show that every even number can be factored as a product of E primes.

Answer: We start by looking at ordinary numbers. Say we have a number z, we would factor it like:

if $z=2 \rightarrow$ prime $(2\cdot1)$
$z=3 \rightarrow$ prime $(3\cdot1)$
$z=4 \rightarrow$ product of primes (2^2)
$z=5 \rightarrow$ prime $(5\cdot1)$
$z=6 \rightarrow$ product of primes $(2\cdot3)$

We can say that the concept of product of primes is true where $z \leq 6$.

Now we look at even number z's in the "E-zone."

when $z=2 \rightarrow$ E prime - yes
$z=4 \rightarrow$ E prime - product of E primes (2^2)
$z=6 \rightarrow$ E prime - yes
$z=8 \rightarrow$ E prime - product of E primes $(2\cdot4)$
$z=10 \rightarrow$ E prime - yes

So our hypothesis is true that all even numbers in the "E-zone" will be a product of "E-primes" when $z \leq 10$.

Now we will assume true for $n \le z-2$ and look at z. If z if prime then we are done. However, we can show $z = ab$ for integers ($2 < a, b$) by induction (hence $a, b \le z-2$).

a and b are both products of "E-primes"

$\therefore$ z is a product of "E-primes".

if $a=2$ b odd, done, otherwise both a, b are even

C) We saw that 180 has 3 different factorizations as a product of E-primes. Find the smallest number that has 2 different factorizations as a product of E-primes. Is 180 the smallest with three? Find the smallest number with four factorizations.

<u>Two Factorizations:</u>

$4 = 2 \cdot 2$ (1)

$8 = 2 \cdot 2 \cdot 2$ (1)

$12 = 6 \cdot 2$ (1)

$16 = 2 \cdot 2 \cdot 2 \cdot 2$ (1)

$20 = 2 \cdot 10$ (1)

$24 = 2 \cdot 2 \cdot 6$ (1)

$28 = 2 \cdot 14$

$32 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ (1)

$\boxed{36 = 2 \cdot 18 = 6 \cdot 6}$ (2)

$\therefore$ 36 is the lowest number with 2 different factorizations of "E-Primes"

<u>Three Factorizations:</u>

After our discovery that 36 was the lowest number with two factorizations, we tested all multiples of 36 until we found a number with three factorizations

$72 = 6 \cdot 2 \cdot 6 = 18 \cdot 2 \cdot 2$ (2)

$108 = 2 \cdot 54 = 6 \cdot 18$ (2)

$144 = 2 \cdot 2 \cdot 6 \cdot 6 = 2 \cdot 2 \cdot 2 \cdot 18$ (2)

$\boxed{180 = 6 \cdot 30 = 10 \cdot 18 = 2 \cdot 90}$ (3)

$\therefore$ We conclude that 180 is the lowest number with 3 factorizations of "E-Primes"

Four Factorizations:

Using the same concept as before, since 36 is the lowest number with two factorizations and 180 is the lowest with three, we considered that the lowest number with four factorizations would be the LCM of those two numbers.

NOTE: In the "E-zone", the LCM of 36 and 180 is 360, because 5, the actual LCM, does not exist in this "world of numbers"

upon testing 360, we found that it does, in fact, have four different factorizations

$$\begin{aligned}360 &= 10\cdot6\cdot6\\ &= 10\cdot18\cdot2\\ &= 2\cdot6\cdot30\\ &= 2\cdot2\cdot90\end{aligned}$$

Tests on 216, 252, 288, and 324 (the multiples of 36 between 180 and 360) did not result in a number with four factorizations. This further proves that 360 must be the smallest number w/ 4 factorizations d.) All even numbers that have only one factorization have a common identity.

When the number is divided by 2, it will result in either:

Ⓐ another E-Prime

Ⓑ another even number that has only one factorization.

Ex:

$$\left.\begin{aligned} 4 &= 2\cdot 2 \\ 12 &= 2\cdot 6 \end{aligned}\right\}\ 2, 6 \text{ are E-Primes}$$

$$\left.\begin{aligned} 16 &= 2\cdot 8 \\ &= 2\cdot(2\cdot 2\cdot 2) \\ 32 &= 2\cdot 16 \\ &= 2\cdot(2\cdot 2\cdot 2\cdot 2) \\ 64 &= 2\cdot 32 \\ &= 2\cdot(2\cdot 2\cdot 2\cdot 2\cdot 2) \end{aligned}\right\}\ \text{8, 16, 32 all only have one factorization}$$

$$\left.\begin{aligned} 52 &= 2\cdot 26 \\ 92 &= 2\cdot 46 \end{aligned}\right\}\ 26, 46 \text{ are E-Primes}$$

Extension:

What about a number with five E-prime factorizations?

1620	1620	1620	1620	1620
$2 \cdot 810$	$8 \cdot 270$	$162 \cdot 10$	$18 \cdot 90$	$30 \cdot 54$

1620 is a number with five E-prime factorizations.