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Due: April 3, 2006  
Problem 6.6

a) The equation  $3x + 5y = 4$  has no solutions with  $x \geq 0$  and  $y \geq 0$ . First of all, 5 is greater than 4, so therefore  $y$  would have to be 0. Then we have the equation  $3x = 4$ . There is no integer multiple of 3 that is equal to 4.

b) We calculated by hand and we discovered the values that are possible. These integer values include: 0, 3, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15... and from here on, every integer was included.

**Conjecture:** All integer values are possible except for 1, 2, 4, and 7.

**Proof:**

```
> sort([seq(seq(3*x+5*y, x=0..6), y=0..6)]);  
[0, 3, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 15, 16, 17, 18, 18, 19, 20, 20, 21, 22, 23, 23, 24, 25, 25, 26, 27, 28, 28,  
29, 30, 30, 31, 32, 33, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 45, 48]
```

c)

(i)  $(a, b) = (3, 7)$

```
> sort([seq(seq(3*x+7*y, x=0..6), y=0..6)]);  
[0, 3, 6, 7, 9, 10, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36,  
37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 50, 51, 53, 54, 57, 60]
```

**The largest number is 11.**

(ii)  $(a, b) = (5, 7)$

```
> sort([seq(seq(5*x+7*y, x=0..6), y=0..6)]);  
[0, 5, 7, 10, 12, 14, 15, 17, 19, 20, 21, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41,  
42, 43, 44, 45, 46, 47, 48, 50, 51, 52, 53, 55, 57, 58, 60, 62, 65, 67, 72]
```

**The largest number is 23.**

(iii)  $(a, b) = (4, 11)$

```
> sort([seq(seq(4*x+11*y, x=0..10), y=0..10)]);  
[0, 4, 8, 11, 12, 15, 16, 19, 20, 22, 23, 24, 26, 27, 28, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44,  
45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72,  
73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100,  
101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 122,  
123, 124, 126, 127, 128, 130, 131, 134, 135, 138, 139, 142, 146, 150]
```

**The largest number is 29.**

**d) Conjectural Formula:  $a*b - (a+b)$**

**Check your conjecture with at least two more values.**

Let  $(a, b) = (4, 13)$

$$4(13) - (4+13) = 51 - 17 = 35$$

```
> sort([seq(seq(4*x+13*y, x=0..10), y=0..10)]);  
[0, 4, 8, 12, 13, 16, 17, 20, 21, 24, 25, 26, 28, 29, 30, 32, 33, 34, 36, 37, 38, 39, 40, 41, 42, 43, 45, 46, 47, 49,  
50, 51, 52, 53, 54, 55, 56, 58, 59, 60, 62, 63, 64, 65, 66, 67, 68, 69, 71, 72, 73, 75, 76, 77, 78, 79, 80, 81,  
82, 84, 85, 86, 88, 89, 90, 91, 92, 93, 94, 95, 97, 98, 99, 101, 102, 103, 104, 105, 106, 107, 108, 110, 111,  
112, 114, 115, 116, 117, 118, 119, 120, 121, 123, 124, 125, 127, 128, 129, 130, 131, 132, 133, 134, 136,  
137, 138, 140, 141, 142, 144, 145, 146, 149, 150, 153, 154, 157, 158, 162, 166, 170]
```

Let  $(a, b) = (22, 23)$

$$22(23) - (22+23) = 483 - 44 = 439$$

```
> sort([seq(seq(21*x+23*y, x=0..25), y=0..25)]);  
[0, 21, 23, 42, 44, 46, 63, 65, 67, 69, 84, 86, 88, 90, 92, 105, 107, 109, 111, 113, 115, 126, 128, 130, 132, 134,  
136, 138, 147, 149, 151, 153, 155, 157, 159, 161, 168, 170, 172, 174, 176, 178, 180, 182, 184, 189, 191,  
193, 195, 197, 199, 201, 203, 205, 207, 210, 212, 214, 216, 218, 220, 222, 224, 226, 228, 230, 231, 233,  
235, 237, 239, 241, 243, 245, 247, 249, 251, 252, 253, 254, 256, 258, 260, 262, 264, 266, 268, 270, 272,  
273, 274, 275, 276, 277, 279, 281, 283, 285, 287, 289, 291, 293, 294, 295, 296, 297, 298, 299, 300, 302,  
304, 306, 308, 310, 312, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 325, 327, 329, 331, 333, 335,  
336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 348, 350, 352, 354, 356, 357, 358, 359, 360, 361,  
362, 363, 364, 365, 366, 367, 368, 369, 371, 373, 375, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386,  
387, 388, 389, 390, 391, 392, 394, 396, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410,  
411, 412, 413, 414, 415, 417, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433,  
434, 435, 436, 437, 438, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455,  
456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476,  
477, 478, 479, 480, 481, 482, 483, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496,  
497, 498, 499, 500, 501, 502, 503, 504, 504, 505, 506, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515,  
516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 525, 526, 527, 527, 528, 529, 529, 530, 531, 532, 533,  
534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 548, 549, 550, 550, 551, 552,
```

**e) Prove that the formula is correct.**

**Proof by example using Maple:**

Let  $(a, b) = (7, 15)$

$$7(15) - (7+15) = 83$$

```
> sort([seq(seq(7*x+15*y, x=0..10), y=0..10)]);  
[0, 7, 14, 15, 21, 22, 28, 29, 30, 35, 36, 37, 42, 43, 44, 45, 49, 50, 51, 52, 56, 57, 58, 59, 60, 63, 64, 65, 66, 67,  
70, 71, 72, 73, 74, 75, 78, 79, 80, 81, 82, 85, 86, 87, 88, 89, 90, 93, 94, 95, 96, 97, 100, 101, 102, 103,  
104, 105, 108, 109, 110, 111, 112, 115, 116, 117, 118, 119, 120, 123, 124, 125, 126, 127, 130, 131, 132,  
133, 134, 135, 138, 139, 140, 141, 142, 145, 146, 147, 148, 149, 150, 153, 154, 155, 156, 157, 160, 161,  
162, 163, 164, 168, 169, 170, 171, 175, 176, 177, 178, 183, 184, 185, 190, 191, 192, 198, 199, 205, 206,  
213, 220]
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Let  $(a, b) = (31, 121)$   
 $31(121) - (3+121) = 869$

```
> sort([seq(seq(30*x+31*y, x=0..30), y=0..30)]);  
[0, 30, 31, 60, 61, 62, 90, 91, 92, 93, 120, 121, 122, 123, 124, 150, 151, 152, 153, 154, 155, 180, 181, 182, 183,  
184, 185, 186, 210, 211, 212, 213, 214, 215, 216, 217, 240, 241, 242, 243, 244, 245, 246, 247, 248, 270,  
271, 272, 273, 274, 275, 276, 277, 278, 279, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 330,  
331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369,  
370, 371, 372, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 420, 421, 422, 423,  
424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459,  
460, 461, 462, 463, 464, 465, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494,  
495, 496, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 540,  
541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 570, 571, 572,  
573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 600, 601, 602, 603,  
604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 630, 631, 632, 633,  
634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 660, 661, 662,  
663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 690,  
691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711,  
712, 713, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738,  
739, 740, 741, 742, 743, 744, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764,  
765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789,  
790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 810, 811, 812, 813,  
814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834,  
835, 836, 837, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857,  
858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879,
```

f) What is the largest number that is not of the form  $6x + 10y + 15z$  with nonnegative  $x, y, z$ ?

```
> sort([seq(seq(seq(6*x+10*y+15*z, x=0..6), y=0..6), z=0..6)]);  
[0, 6, 10, 12, 15, 16, 18, 20, 21, 22, 24, 25, 26, 27, 28, 30, 30, 30, 31, 32, 33, 34, 35, 36, 36, 36, 37, 38, 39, 40,  
40, 40, 41, 42, 42, 43, 44, 45, 45, 45, 46, 46, 46, 47, 48, 48, 49, 50, 50, 50, 51, 51, 51, 52, 52, 53, 54, 54,  
55, 55, 55, 56, 56, 56, 57, 57, 58, 58, 59, 60, 60, 60, 60, 60, 61, 61, 61, 62, 62, 63, 63, 64, 64, 65, 65, 65,  
66, 66, 66, 66, 66, 67, 67, 68, 68, 69, 69, 70, 70, 70, 70, 71, 71, 71, 72, 72, 72, 73, 73, 74, 74, 75, 75, 75,  
75, 75, 76, 76, 76, 76, 77, 77, 78, 78, 78, 79, 79, 80, 80, 80, 80, 81, 81, 81, 81, 81, 82, 82, 83, 83, 84, 84,  
84, 85, 85, 85, 85, 86, 86, 86, 86, 87, 87, 87, 88, 88, 89, 89, 90, 90, 90, 90, 90, 91, 91, 91, 91, 92, 92,  
93, 93, 93, 94, 94, 95, 95, 95, 95, 96, 96, 96, 96, 96, 96, 97, 97, 98, 98, 99, 99, 99, 100, 100, 100, 100,  
101, 101, 101, 101, 102, 102, 102, 103, 103, 104, 104, 105, 105, 105, 105, 105, 106, 106, 106, 106, 107,  
107, 108, 108, 108, 109, 109, 110, 110, 110, 110, 111, 111, 111, 111, 111, 112, 112, 113, 113, 114, 114,  
114, 115, 115, 115, 116, 116, 116, 116, 117, 117, 118, 118, 119, 119, 120, 120, 120, 120, 120, 121, 121,  
121, 122, 122, 123, 123, 124, 124, 125, 125, 125, 126, 126, 126, 126, 126, 127, 128, 128, 129, 129, 130,  
130, 130, 131, 131, 131, 132, 132, 133, 134, 134, 135, 135, 135, 136, 136, 136, 137, 138, 138, 139, 140,  
140, 140, 141, 141, 141, 142, 143, 144, 144, 145, 146, 146, 146, 147, 148, 149, 150, 150, 150, 151, 152,  
153, 154, 155, 156, 156, 156, 158, 159, 160, 161, 162, 164, 165, 166, 168, 170, 171, 174, 176, 180, 180]
```

The greatest number that is not of this form is 29.