5.4 Final Draft

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A number L is called a common multiple of m and n if both m and n divide L. The smallest such L is called the least common multiple of m and n and is denoted by $Lcm(m,n)$. For example $Lcm(3,7)=21$ $Lcm(12,66)=132$

a. Find the following least common multiples.
We computed them on maple
(i) $Lcm(8,12) \rightarrow 24$
(ii) $Lcm(20,30) \rightarrow 60$
(iii) $Lcm(51,68) \rightarrow 204$
(iv) $Lcm(23,18) \rightarrow 414$

b. For each of the Lcm's that you computed in a, compare the value of $Lcm(m,n)$ to the values of m, n and the gcd of (m,n) Try to find a relationship.

(i) $Lcm = 24$
$m = 8$
$n = 12$
gcd of $(8,12) = 4$

(ii) $Lcm = 60$
$m = 20$
$n = 30$
gcd of $(20,30) = 10$

(iii) $Lcm = 204$
$m = 51$
$n = 68$
gcd of $(51,68) = 17$

(iv) $Lcm = 414$
$m = 23$
$n = 18$
gcd of $(23,18) = 1$

From the observations we discovered that the gcd's from i, ii are both even and that the Lcm's of both are $Lcm = (3m, 2n)$

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To show or observation we see that for i : LCM = $3 \cdot 8$ or $2 \cdot 12$
both equal 24

For iii and iv we noted the gcd for both were odd
(iii = 17, iv = 23)
and that iii had a LCM = 4m or 3n
Finally iv had an LCM of 414 = $23 \cdot 18$

C. Prove the relationship for m and n.

$m = kl$
$n = ks$
$LCM = \frac{m \cdot n}{GCD}$

We know lsk is a common multiple of m and n, claim is the least common multiple.

Let's take P as another common multiple
$P = a \cdot m = b \cdot n$
so $a \cdot k \cdot l = b \cdot k \cdot s$

So if ~~L~~/P then $L \leq P$ and therefore L is the least common multiple.

So we have $L = kls$ so $\overset{L}{kls} / \overset{P}{akl}$ why does $s | a$?

Since L divides P we have L being the least common multiple

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c cont. So since $P = a \cdot m = b \cdot n$

$$P = ak\ell = bks$$
$$\Rightarrow a\ell = bs$$

since $\gcd(\ell, s) = 1$ so $\ell / bs \Rightarrow \ell / b$
so $b = c\ell$ for some c.

thus $P = c\ell ks$ and $L = \ell ks$ so L / P
and so $L \leq P$ ✓

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d. Using maple we found the GCD = 541 and the formula we use is $LCM\left(m\left(\frac{n}{GCD}\right), n\left(\frac{m}{GCD}\right)\right)$ so $LCM(301337, 307829)$

$$\rightarrow LCM\left(301337\left(\frac{307829}{541}\right), 307829\left(\frac{301337}{541}\right)\right)$$

$$= 171460753$$

e. We know that M and N must be multiples of 18. M must be equal to 18 because any other multiple can result in a different GCD.

ex) $GCD(36, 72) = 36$

Taking m to be 18, n must be equal to 720. If n is any other multiple of 18, that larger number would be the LCM of the two numbers because it is a multiple of 18. Therefore $m = 18$ and $n = 720$

Extension: Let's take the gcd of the numbers in d and see if there's a relationship between numbers - maybe both odd or even??

$gcd(301337, 307829) = 541$ → found on maple and known from above

We see that both m and n odd gave an odd. Also that both the LCM and GCD are odd.

```
>
```

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Exercise 5.4

part a

```
> restart;
> ilcm(8, 12);
```

24

```
> ilcm(20, 30);
```

60

```
> ilcm(51, 68);
```

204

```
> ilcm(23, 18);
```

414

```
>
```

part b

```
> igcd(8, 12);
```

4

```
> igcd(20, 30);
```

10

```
> igcd(51, 68);
```

17

```
> igcd(23, 18);
```

1

```
>
```

part d

```
> igcd(301337, 307829);
```

541

```
> ilcm(301337, 307829);
```

171460753