A number \( L \) is called a common multiple of \( m \) and \( n \) if both \( m \) and \( n \) divide \( L \). The smallest such \( L \) is called the least common multiple of \( m \) and \( n \) and is denoted by \( \text{lcm}(m, n) \). For example \( \text{lcm}(3, 7) = 21 \), \( \text{lcm}(12, 16) = 48 \).

a. Find the following least common multiples.

We computed them on maple.

(i) \( \text{lcm}(8, 12) = 24 \)
(ii) \( \text{lcm}(20, 30) = 60 \)
(iii) \( \text{lcm}(5, 108) = 204 \)
(iv) \( \text{lcm}(23, 18) = 414 \)

b. For each of the \( \text{lcm} \)'s that you computed in a, compare the value of \( \text{lcm}(m, n) \) to the values of \( m, n \) and the \( \text{gcd} \) of \( (m, n) \). Try to find a relationship.

(i) \( \text{lcm} = 24 \)
   \[ \begin{align*}
   m &= 8 \\
   n &= 12 \\
   \text{gcd} \text{ of } (8, 12) &= 4
   \end{align*} \]

(ii) \( \text{lcm} = 60 \)
   \[ \begin{align*}
   m &= 20 \\
   n &= 30 \\
   \text{gcd} \text{ of } (20, 30) &= 10
   \end{align*} \]

(iii) \( \text{lcm} = 204 \)
   \[ \begin{align*}
   m &= 51 \\
   n &= 68 \\
   \text{gcd} \text{ of } (51, 68) &= 17
   \end{align*} \]

(iv) \( \text{lcm} = 414 \)
   \[ \begin{align*}
   m &= 23 \\
   n &= 18 \\
   \text{gcd} \text{ of } (23, 18) &= 1
   \end{align*} \]

From these observations we discovered that the \( \text{gcd} \)'s from i, ii are both even, and that the \( \text{lcm} \)'s of both are \( \text{lcm} = (3m, 2n) \).
To show our observation we see that for i: LCM = 3.8 or 3.12 both equal 24

For iii and iv we noted the gcd for both were odd
\[ \text{gcd}(11, 7) = 1, \text{gcd}(23, 12) = 1 \]
and that iv had a LCM = 4m or 3n.
Finally iv had an LCM of \( 4 \times 14 = 23.18 \)

2. Prove the relationship for m and n.

\[ \frac{m}{k} = \frac{n}{l} \]
\[ \text{LCM} = \frac{m \cdot n}{\text{GCD}} \]

We know \( k \cdot l \) is a common multiple of \( m \) and \( n \), claim is the least common multiple.

Let's take \( P \) as another common multiple
\[ P = a \cdot m = b \cdot n \]
so \( a \cdot k \cdot l = b \cdot k \cdot s \)

So if \( P \) then \( L \leq P \) and therefore \( L \) is the least common multiple.

So we have \( L = k \cdot s \), so \( k \cdot s \) could be the least common multiple.

Since \( L \) divides \( P \) we have \( L \) being the least common multiple.
c cont. So since $P = \alpha \cdot m = b \cdot n$

$$P = akl = bks$$

$$\Rightarrow a \ell = bs$$

since $gcd(l, s) = 1$ so $\ell/bs \Rightarrow \ell/b$

so $b = c\ell$ for some $c$.

Thus $P = clks$ and $L = lks$ so $\ell/P$

and so $L \leq P \sqrt{ }$
d. Using maple we found the GCD = 541 and the formula we use is 

\[ \text{LCM} \left( \frac{m}{\text{GCD}}, \frac{n}{\text{GCD}} \right) \] 

so \[ \text{LCM}(301337, \frac{307829}{541}) \]

\[ \to \text{LCM}(301337, 307829 \cdot \frac{301337}{307829}) \]

\[ = 171440753 \]

C. We know that \( M \) and \( N \) must be multiples of 18. \( M \) must be equal to 18, because any other multiple can result in a different GCD.

So \( \text{GCD}(34, 22) = 34 \) 
Taking \( M \) to be 18, \( n \) must be equal to 720. If \( n \) is any other multiple of 18, that larger number would be the LCM of the two numbers because it is a multiple of 18. Therefore \( m = 18 \) and \( n = 720 \).

Extension: Let's take the gcd of the numbers in d and see if there's a relationship between numbers -- maybe both odd or even?

\[ \text{gcd}(301337, 307829) = 541 \] 

\[ \to \text{found on maple and known from above} \]

We see that both \( M \) and \( n \) odd gave an odd. Also that both the LCM and GCD are odd.
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Exercise 5.4

part a

> restart;

> ilcm(8, 12);

> ilcm(20, 30);

> ilcm(51, 68);

> ilcm(23, 18);

> part b

> igcd(8, 12);

> igcd(20, 30);

> igcd(51, 68);

> igcd(23, 18);

> part d

> igcd(301337, 307829);

> ilcm(301337, 307829);