5.4 Final Draft:

a) i: \( \text{LCM} (8, 12) = 24 \)
   ii: \( \text{LCM} (20, 30) = 60 \)
   iii: \( \text{LCM} (51, 68) = 204 \)
   iv: \( \text{LCM} (23, 18) = 414 \)

b) \( \text{LCM} (m, n) = \frac{m \cdot n}{\text{gcd}(m, n)} \)

c) We let \( K = \text{gcd}(m, n) \), then we can rewrite \( m \) and \( n \) as a product of the gcd and some other number.

So: \( m = kx \) and \( n = ky \) and the \( \text{gcd}(x, y) = 1 \) since \( K \) is the largest number that can divide both \( m \) and \( n \).

We can substitute these values into our formula from part b.

\[ \text{LCM} = \frac{mn}{\text{gcd}(m,n)} \rightarrow \frac{(kx)(ky)}{K} \rightarrow \frac{K^2xy}{K} \rightarrow Kxy \]
Thus, since \( x \) and \( y \) have a \( \text{gcd} \) of 1, and \( K \) is the \( \text{gcd}(m,n) \), then \( kxy \) is the smallest number that \( m \) and \( n \) can both go into.

This proves that our formula \( \left( \frac{mn}{\text{gcd}(m,n)} \right) \) does in fact give the \( \text{LCM} \) of \( (m,n) \).

d) \( \text{LCM}(301337, 307829) \):

\[
\begin{align*}
\text{gcd:} & \quad 307829 = 301337 + 6492 \\
& \quad 301337 = (46 \cdot 6492) + 2705 \\
& \quad 6492 = (2 \cdot 2705) + 1082 \\
& \quad 2705 = (2 \cdot 1082) + 541 \\
& \quad 1082 = (2 \cdot 541) + 0 \\
\end{align*}
\]

\[
\text{gcd} = 541
\]

\[
\frac{301337 \cdot 307829}{541} = (171,460,753)
\]

e) \( \text{LCM}(m,n) = \frac{m \cdot n}{\text{gcd}(m,n)} \rightarrow 720 = \frac{(m \cdot n)}{18} \) if \( (m,n) = 12960 \)

- If \( (m,n) = 12960 \) then \( m = 18, \ n = 720 \)
  \( m = 90, \ n = 144 \)
Part C add on:

Let \( P \) equals some common multiple

\[
P = am = bm
\]

\[
\Rightarrow P = a \cdot k \cdot x = b \cdot k \cdot y
\]

\[
ax = by \quad \text{gcd}(x, y) = 1
\]

\[
\Rightarrow x | by \Rightarrow x | b
\]

\[
\Rightarrow b = cx \text{ for some } c
\]

Thus \( P = cxky \)

and \( L = xky \)

\[
\Rightarrow L | P \text{ and so } L \leq P
\]

therefore \( L \) is the least common multiple.
Extension:

Find the \( \text{LCM}(1280, 2006) \)

\[ \gcd(1280, 2006): \]

\[
\begin{align*}
2006 &= 1280 + 726 \\
1280 &= 726 + 554 \\
726 &= 554 + 172 \\
554 &= (172 \cdot 3) + 38 \\
172 &= (38 \cdot 4) + 20 \\
38 &= (20 \cdot 1) + 18 \\
20 &= 18 + 2 \\
18 &= (2 \cdot 9) + 0
\end{align*}
\]

\[ \gcd = 2 \]

\[ \text{LCM} = \frac{(1280 \cdot 2006)}{\gcd} = \frac{(1280 \cdot 2006)}{2} = 1,283,840 \]