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5.4

Final Draft:

- a) i: $\text{LCM}(8, 12) = 24$
ii: $\text{LCM}(20, 30) = 60$
iii: $\text{LCM}(51, 68) = 204$
iv: $\text{LCM}(23, 18) = 414$

b) $\text{LCM}(m, n) = \frac{m \cdot n}{\text{gcd}(m, n)}$

c) We let $k = \text{gcd}(m, n)$, then we can rewrite m and n as a product of the gcd and some other number.

So: $m = kx$ and $n = ky$
and the $\text{gcd}(x, y) = 1$ since k is the largest number that can divide both m and n .

We can substitute these values into our formula from part b.

$$\text{LCM} = \frac{mn}{\text{gcd}(m, n)} \rightarrow \frac{(kx)(ky)}{k} \rightarrow \frac{k^2 xy}{k} \rightarrow kxy$$

- Thus, since x and y have a gcd of 1 and K is the gcd(m, n) then Kxy is the smallest number that m and n can both go into.

* see next sheet *

- This proves that our formula $\left(\frac{mn}{\text{gcd}(m,n)}\right)$ does in fact give the LCM of (m, n).

d) LCM(301337, 307829):

$$\text{gcd: } 307829 = 301337 + 6492$$

$$301337 = (46 \cdot 6492) + 2705$$

$$6492 = (2 \cdot 2705) + 1082$$

$$2705 = (2 \cdot 1082) + \textcircled{541}$$

$$1082 = (2 \cdot 541) + 0$$

$$\text{gcd} = 541$$

$$\text{LCM}(301337, 307829) = \frac{(301337 \cdot 307829)}{541} = \textcircled{171,460,753}$$

$$\text{e) } \text{LCM}(m, n) = \frac{m \cdot n}{\text{gcd}(m, n)} \rightarrow 720 = \frac{(m \cdot n)}{18} \text{ if } (m \cdot n) = 12960$$

$$\text{- if } (m \cdot n) = 12960 \text{ then } \begin{array}{l} m = 18, n = 720 \\ m = 90, n = 144 \end{array}$$

Part C add on:

Let

P equals some common multiple

$$P = am = bn$$

$$\text{so } P = akx = bky$$

$$ax = by$$

$$\text{gcd}(x, y) = 1$$

$$\text{so } x | by \Rightarrow x | b$$

so $b = cx$ for some c .

$$\text{Thus } P = cxky$$

$$\text{and } L = xky$$

$$\text{so } L | P \text{ and so } L \leq P$$

therefore L is the least common multiple.

Extension:

Find the LCM (1280, 2006)

- gcd (1280, 2006):

$$2006 = 1280 + 726$$

$$1280 = 726 + 554$$

$$726 = 554 + 172$$

$$554 = (172 \cdot 3) + 38$$

$$172 = (38 \cdot 4) + 20$$

$$38 = (20 \cdot 1) + 18$$

$$20 = 18 + \textcircled{2} \leftarrow$$

$$18 = (2 \cdot 9) + 0 \rightarrow$$

$$\text{gcd} = 2$$

$$\text{LCM} = \frac{(1280, 2006)}{\text{gcd}} = \frac{(1280 \cdot 2006)}{2} = \textcircled{1,283,840}$$