

Ben Marlow (Final Draft)
 Thomas Raymond (Rough Draft)
 John Lucy (Notes)
 Lauren Rizzotti (Notes)
 Abby Bayer (Presentation)

3.1 Group B

A). If u and v have a common factor, it will not be a primitive Pythagorean triple, because if the values are subtracted or added, then the resulting answer will always be a common factor of both u and v . This will cause a , b and c to always have a common factor of both u and v .

$$u = dk$$

$$v = dl$$

$$a = u^2 - v^2$$

$$a = (dk)^2 - (dl)^2$$

$$a = d^2(k^2 - l^2)$$

$$b = 2uv$$

$$b = 2(dk)(dl)$$

$$b = 2d(k)(l)$$

$$c = u^2 + v^2$$

$$c = (dk)^2 + (dl)^2$$

$$c = d^2(k^2 + l^2)$$

As you can easily see, a factor of d is able to be brought out of each of the a , b , and c equations for Pythagorean triples.

B).

$$u = 13$$

$$v = 7$$

$$a = u^2 - v^2 = 13^2 - 7^2 = 120$$

$$b = 2uv = 2(13)(7) = 182$$

$$c = u^2 + v^2 = 13^2 + 7^2 = 218$$

It is clearly visible that there is a common factor of 2 which can be factored out of this Pythagorean triples.

C). See Attached Maple

D). Using our table, we are able to conclude that, both u and v cannot have common factors, when either u or v is odd then a & c will be odd (which will allow

the triple to be primitive) and v cannot be zero. By following these three conditions, the Pythagorean triple which is being calculated will be primitive. If $u+v$ is even, then the resulting sum is not primitive, and if $u+v$ is odd, then the resulting sum is primitive. The final fact is that u must be greater than v .

 If $u = 2c$ and $v = 2d+1$, then

$$a = u^2 - v^2$$

$$a = (2c)^2 - (2d+1)^2$$

$$a = 4c^2 - (4d^2 + 4d + 1)$$

$$a = 4(c^2 - d^2 - d) - 1$$

From this example, a must be odd because anything squared will give an even number, and by adding 1, the number then results to being odd.

$$b = 2uv$$

$$b = 2(2c)(2d+1)$$

$$b = 2(4cd + 2c)$$

$$b = 8cd + 4c$$

$$b = 4c(2d+1)$$

From this example, b must be even because anything squared plus some number multiplied by an even number will always give a sum which is even.

$$c = u^2 + v^2$$

$$c = (2c)^2 + (2d+1)^2$$

$$c = 4c^2 + 4d^2 + 4d + 1$$

$$c = 4(c^2 + d^2 + d) + 1$$

From this example, c must be odd because $4(c^2 + d^2 + d)$ equals an even number. So when we add 1 to this, it will always give us an odd number.

Now, if we change the values so that u is an odd number and v is an even number, it will be shown that the same results take place:

If $u = 2d+1$ and $v = 2c$, then

From this example, a must be odd because anything squared will give an even number, and by adding 1, the number then results to being odd.

$$b = 2uv$$

$$b = 2(2d + 1)(2c)$$

$$b = 2(4cd + 2c)$$

$$b = 8cd + 4c$$

From this example, b must be even because anything squared plus some number multiplied by an even number will always give a sum which is even.

$$c = u^2 + v^2$$

$$c = (2d + 1)^2 + (2c)^2$$

$$c = 4d^2 + 4d + 1 + 4c^2$$

$$c = 4(c^2 + d^2 + d) + 1$$

From this example, c must be odd because $4(c^2 + d^2 + d)$ equals an even number. So when we add 1 to this, it will always give us an odd number.

If both u and v are even, then:

$$u = 2c$$

$$v = 2v$$

$$a = u^2 - v^2$$

$$a = (2c)^2 - (2v)^2$$

$$a = 4c^2 - 4v^2$$

$$a = 4(c^2 - v^2)$$

$$b = 2uv$$

$$b = 2(2c)(2v)$$

$$b = 8(c)(v)$$

$$c = u^2 + v^2$$

$$c = (2c)^2 + (2v)^2$$

$$c = 4c^2 + 4v^2$$

$$c = 4(c^2 + v^2)$$

As is seen, (a, b, c) are all even because there is a common factor of 4 for each of them.

If both u and v are odd, then:

$$u = 2c + 1$$

$$v = 2v + 1$$

$$a = u^2 - v^2$$

$$a = (2c + 1)^2 - (2v + 1)^2$$

$$a = (2c^2 + 4c + 1) - (2v^2 + 4v + 1)$$

$$a = 2c^2 + 4c + 2v^2 + 4v$$

$$b = 2uv$$

$$b = 2(2c + 1)(2v + 1)$$

$$b = 8(c + 2)(v + 2)$$

$$c = u^2 + v^2$$

$$c = (2c + 1)^2 + (2v + 1)^2$$

$$c = (2c^2 + 4c + 1) + (2v^2 + 4v + 1)$$

$$c = 2c^2 + 4c + 2v^2 + 4v + 2$$

As is seen, a, b, and c all have common factors of 2. So, if both u and v are either odd or even, then the resulting Pythagorean triple will have a common factor.

**Group B
Extension**

3.1

What would happen if u did not always have to be greater than v ? Would we still get the same answer?

The answer will be no, we will not get the same answer because, if v could be greater than u , it would give us a negative value on the right side of the list on maple.

The reason why we are getting a negative value is because, when v is greater than u , the output of a has to be a negative number.

For example, if $u = 2$ and $v = 4$, then the values for a will look as follows:

$$a = u^2 - v^2$$

$$a = (2)^2 - (4)^2$$

$$a = 4 - 16$$

$$a = -12$$

As one can see, because the value for v is greater than the value for u , than the value for a will be a negative number. So the answer to the question is no, we will not get the same answer because the value for a will be negative