Chapter 2 Excercise 2.7
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Question : For each primitive Pythagorean triple (*a, b, c*) in the table in this chapter, compute the quantity *2c-2a*. Do these values seem to have some special form? Try to prove that your observation is true for all primitive Pythagorean triples.

When observing the results of calculating 2c-2a, where a, b, and c represent the Pythagorean triples, we found a clear form. When subtracting 2a from 2c, where a is always odd, we found that the result was always a square. Here is a chart of our observations :

a	***b***	***c***
3	4	5
5	12	13
15	8	17
7	24	25
21	20	29
9	40	41
35	12	37

2c	-	***2a***		***Answer***
10	-	6	=	4
26	-	10	=	16
34	-	30	=	4
50	-	14	=	36
58	-	42	=	16
82	-	18	=	64
130	-	66	=	64

$a = 2st$ $b = \frac{s^2 - t^2}{2}$ $c = \frac{s^2 + t^2}{2}$

As shown on the right column, each of the above answers form a square number. We know that every primitive Pythagorean triple, (a, b, c), with a being odd and b being even is explained using the formulas

$$a = st \qquad c = \frac{s^2 + t^2}{2}$$

Therefore we can substitute in

$$2c - 2a$$

$$2\left(\frac{s^2+t^2}{2}\right) - 2(st) = s^2 + t^2 - 2st$$

$$= s^2 - 2st + t^2$$

$$= (s-t)^2$$

As a result we find that 2c-2a will always be a square because $(s - t)^2$ will always be a square.

Extension

If by subtracting 2c-2a you obtain a formula, and find a pattern what happens when you subtract 2c-2b?

a	***b***	***c***
3	4	5
5	12	13
15	8	17
7	24	25
21	20	29
9	40	41
35	12	37

2c	-	***2b***		***Answer***
10	-	8	=	2
26	-	24	=	2
34	-	16	=	18
50	-	48	=	2
58	-	40	=	2
82	-	80	=	2
130	-	24	=	66

Well the answer found showed no pattern, but when you substitute again you get

$$2c - 2b$$

$$2\left(\frac{s^2+t^2}{2}\right) - 2\left(\frac{s^2-t^2}{2}\right)$$

$$= s^2+t^2-s^2+t^2$$

$$= 2t^2$$

Therefore all we can conclude is that the answer will be a multiple of 2? Does this mean anything more?