Group B
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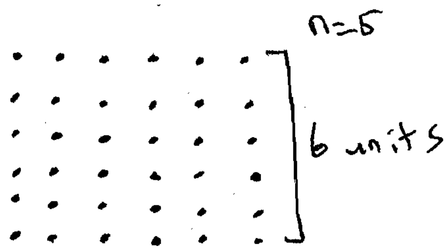
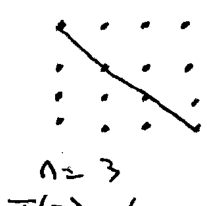
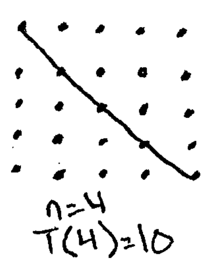
Triangular Numbers

Concerning triangular numbers, we showed geometrically why the recursive and closed formulas worked. To do this, we had to first design square boxes of dots (or circles). The length of each side was to be one more than the value which is being found in terms of n. For example, if you want to find the 5th triangular number, n = 5, then the length of the sides of the box would be 6 units, or dots, long.



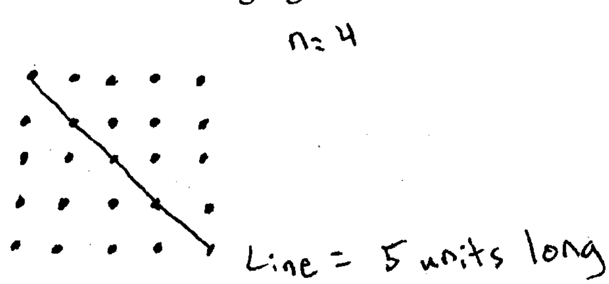
We do this because we then split the box in half to get our desired triangle, and if we hadn't added the extra set of dots then we would be cutting out part of the required triangle. Then, to find the triangular number, we add up all the dots in the resulting triangle. It turns out, because of the nature of the triangles that are created by splitting boxes in half, that to get a new triangular number you add the value of n to the value of n − 1. So, $T(n) = T(n-1) + n$. For example, $T(4) = T(3) + 4$, or $T(4) = 6 + 4 = 10$.





To get to the closed formula of $T(n) = [n(n+1)]/2$, we have to think of the triangles and boxes in terms of areas. The area of the box is $(n+1)^2$, but since when we cut the box in half we are subtracting n + 1 dots, we have to do the following algebra:

$(n+1)^2 = n^2 + 2n + 1$
$n^2 + 2n + 1 - (n+1) = n^2 + n$
$n^2 + n = n(n+1)$



Then, when we have $n(n + 1)$, since we're cutting the box in half, we have to divide by 2. And therefore, we get, as the closed formula, $[n(n + 1)]/2$.

Our extension deals with taking the area of a triangle and finding the difference between its area and the actual triangular number for a certain value of n. We wanted to know if there was some correlation between the areas of the triangles and the triangular number. Given a value for n, which is the base of the triangle, one can find the area of this triangle by ½(b)(h), which gives you $n^2/2$. When the formula for finding triangular numbers, $[n(n + 1)]/2$, is broken down and multiplied out, you get $(n^2/2) + (n/2)$.

When this new formula is considered, the difference between the area of the triangle and the triangular number is $n/2$. For example, if $n = 6$, the triangular number is 21, and when we find the area using $n^2/2$, we get 18. The difference between these is 3, and $n/2 = 3$. Take another example, $n = 8$. Triangular number is 36, area is 32, $n/2 = 4$, and $36 - 4 = 32$. It works for odd numbers as well, though it gets a little messy. If $n = 7$, then the triangular number is 28, area is 24.5, $n/2 = 3.5$, and $28 - 3.5 = 24.5$.

This works for any n. What we are saying is that, the relation between the area of the triangle we create and the triangular number is $n^2/2 + n/2$. We get $n^2/2$ by calculating the area of the triangle (1/2(b)(h)), and when we compare it to the multiplied out form of the triangular number formula, we just have to add $n/2$. That is why the connection between the area and the actual triangular number will work out for any value of n.