Section: Chapter 1 Number Shapes
Presented by: Emily Dowd ✓ (done!)
Final Draft: Anthony Alguo
1 Draft: Jon Kaptcinos
Working Notes: Vincent and Claudette

Anthony Alguo
Final
Page 1

Triangular Numbers: Induction

- The two formulas for the sequence: 1, 3, 6, 10, 15, 21, 28, 36 are:
  1) Closed: $T_m = \frac{m(m+1)}{2}$ m starts at 1
  2) recursive: $T_1 = 1$; $T_n = T_{n-1} + n$ n starts at 2

- Through the process of mathematical Induction it is possible to prove these equations to be correct for all cases

- Base Case: let $m=2$
  using a small number in the equation $\frac{n(m+1)}{2}$

  $$T_2 = \frac{2(2+1)}{2} = \frac{2(3)}{2} = \frac{6}{2} = 3$$

  This proves that at spot 2, the value is 3
  → So the formula $T_m = \frac{m(m+1)}{2}$ holds when $m=2$

- Now lets assume the formula holds up to $n-1$, i.e. we substitute $(n-1)$ from the recursive formula for m in the closed formula and simplify

$$T_{n-1} = \frac{(n-1)[(n-1)+1]}{2} = \frac{n^2-n}{2}$$

Final Continued →

Anthony Aliguo
Final
Page 2

So: $T_n = T_{n-1} + n \iff$

$$\frac{n(n+1)}{2} = \frac{n^2-n}{2} + n \iff$$

$$\frac{n^2+n}{2} = \frac{n^2-n}{2} + \frac{2n}{2} \iff$$

$$\frac{n^2+n}{2} = \frac{n^2-n+2n}{2} \iff$$

$$\frac{n^2+n}{2} = \frac{n^2+n}{2} \checkmark$$

- $T_n = \frac{n(n+1)}{2}$

original equation: $\frac{m(m+1)}{2}$

Extension

if $m = n-2$ $\quad \frac{(n-2)[(n-2)+1]}{2}$

$\boxed{\frac{n^2-3n+2}{2}}$ start at 3

$n=3 \rightarrow \frac{3^2-3(3)+2}{2} = 1$ ← still shows pattern of triangular #'s

$n=4 \rightarrow \frac{4^2-3(4)+2}{2} = \frac{6}{2} = 3$

if $m = n-3$ $\quad \frac{(n-3)[(n-3)+1]}{2}$

$\frac{(n-3)(n-2)}{2}$

$\boxed{\frac{n^2-5n+6}{2}}$ start at 4

$n=4 \rightarrow \frac{16-20+6}{2} = \frac{2}{2} = 1$ ← shows pattern of triangular #'s

$n=5 \rightarrow \frac{25-25+6}{2} = \frac{6}{2} = 3$