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Section 1, Problem 1.1

<u>Question</u>: An effective way to find triangular-square numbers?

<u>Given</u>: 1 and 36, the first triangular-square numbers.
Next triangular-square numbers:
..., 1225, 41616, 1413721, 48024900...

<u>Hypothesis</u>: "$K_n = 34(K_{n-1}) - K_{n-2} + 2$" (Recursive formula)
- www.shyamsundergupta.com/triangle.htm

<u>Proof</u>: We were not sure how to prove this formula. An attempt at using induction was made.

<u>Does the Hypothesis fit the data?</u>

$K_1 = 1$
$K_2 = 36$
$K_3 = 34(36) - 1 + 2 = 1225$ ✓
$K_4 = 34(1225) - 36 + 2 = 41616$ ✓
$K_5 = 34(41616) - 1225 + 2 = 1413727$ ✓

Does the recursive formula give us the triangular-square numbers?

Triangular-Square #	Square (n^2)	Triangular ($\frac{m(m+1)}{2}$)
1	$1^2 = 1$	$m = 1$
36	$6^2 = 36$	$m = 8$
1225	$35^2 = 1225$	$m = 49$
41616	$204^2 = 41616$	$m = 288$
1413721	$1189^2 = 1413721$	$m = 1681$
⋮	⋮	⋮

Check the triangular numbers:

$1 = \frac{m(m+1)}{2}$

$2 = m(m+1)$

$m^2 + m - 2 = 0$

$\boxed{m=1}$ $m \neq 2$

$36 = \frac{m(m+1)}{2}$

$72 = m(m+1)$

$m^2 + m - 72 = 0$

$\boxed{m=8}$

$1225 = \frac{m(m+1)}{2}$

$2450 = m(m+1)$

$m^2 + m - 2450 = 0$

$\boxed{m=49}$

$41616 = \frac{m(m+1)}{2}$

$83232 = m(m+1)$

$m^2 + m - 83232 = 0$

$\boxed{m=288}$

$1413721 = \frac{m(m+1)}{2}$

$2827442 = m(m+1)$

$m^2 + m - 2827442 = 0$

$\boxed{m=1681}$

→ We believe that the triangular-square numbers are infinitely many.

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First we looked at the square numbers that also gave us square-triangular numbers. These numbers include the following first few numbers in the sequence:

$$\{1, 6, 35, 204, 1189, 6930, \ldots\}$$

We tried to find a number n that would give the n^{th} term of the sequence of square numbers that would be squared to find the sequence of triangular numbers.

Observations:

$n_1 = 1$	
$n_2 = 6$	$n_2/n_1 = 6/1 = 6$
$n_3 = 35$	$n_3/n_2 = 35/6 = 5.833$
$n_4 = 204$	$n_4/n_3 = 204/35 = 5.829$
$n_5 = 1189$	$n_5/n_4 = 1189/204 = 5.828$
$n_6 = 6930$	$n_6/n_5 = 6930/1189 = 5.828$
$n_7 = 40,391$	$n_7/n_6 = 40,391/6930 = 5.828$
$n_8 = 235,416$	$n_8/n_7 = 235,416/40,391 = 5.828$
$n_9 = 1,372,105$	$n_9/n_8 = 1,372,105/235,416 = 5.828$
$n_{10} = 7,997,214$	$n_{10}/n_9 = 7,997,214/1,372,105 = 5.829$

Hypothesis:
From this data we can give another approximate formula to find the next sequential square number to find the next triangular square number.

$$u_{n+2} = 6_{n+1} - u_n$$

for $u_0 = 1$
$u_1 = 6$

Check: $u_5 = 6930 \stackrel{?}{=} 6(1189) - 204$
$= 6930$ ✓

$u_{10} = 6(7,997,214) - 1,372,105$
$= 46,611,179$ ✓

Idea was created from ideas on the following website:
http://www.courseworkhelp.co.uk/A_Level/Maths/2.htm