So.... the Euclidean algorithm will give you <u>one</u> solution to

$$a \cdot s + b t = \gcd(a, b)$$

But there are others.

How do we get 'em all?

Let $g = \gcd(a,b)$

We want all integer points on the line

$$ax + by = g$$

(We know s, t from the Euclidean Algorithm is on the line) eg. $\gcd(288, 51) = 3$

we want integer solutions to

$$288x + 51y = 3$$

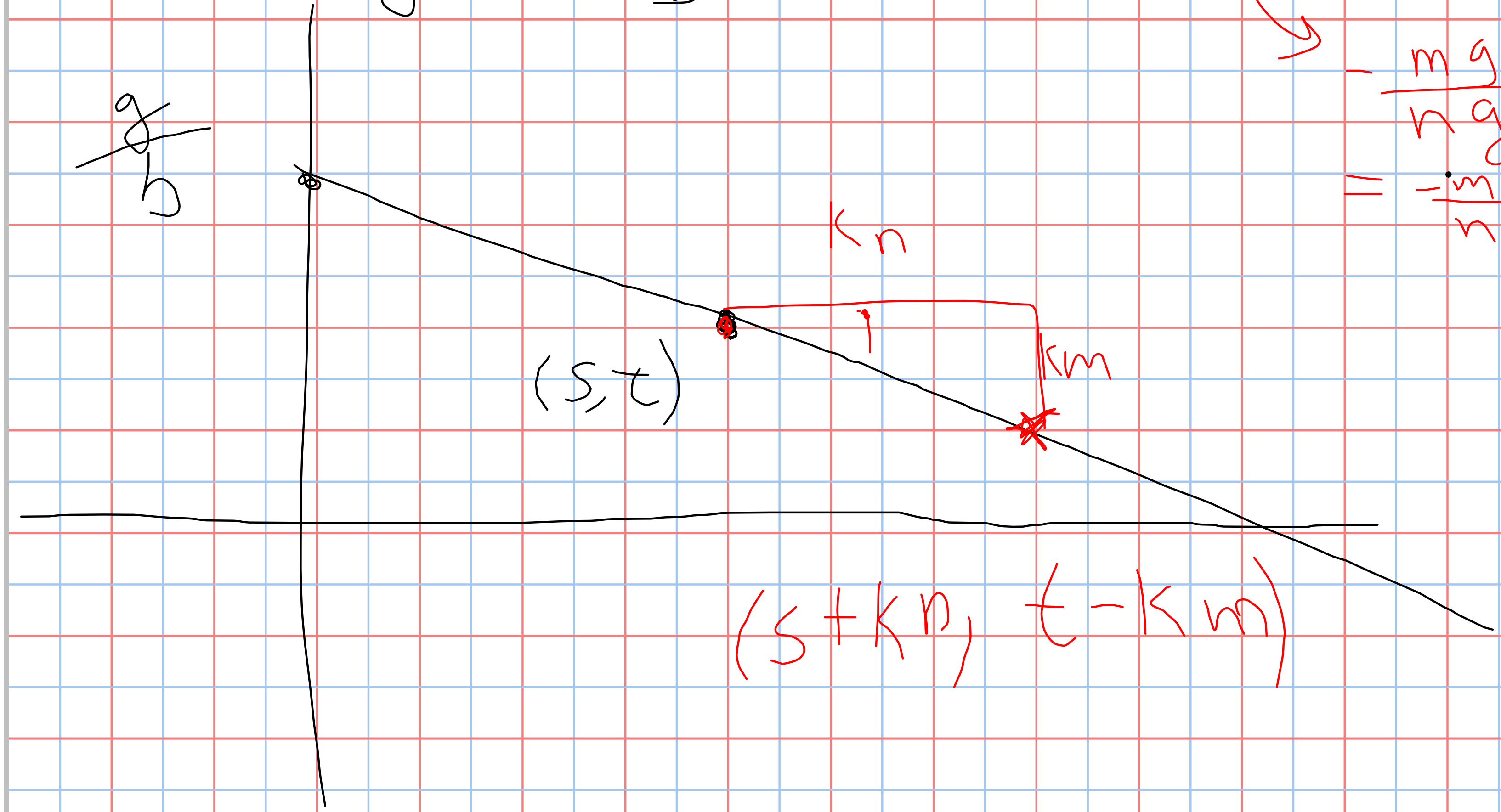
we know $(x, y) = (-3, 17)$

is a solution from the Euclidean algorithm.

The line $ax + by = g$

is $y = \dfrac{g - ax}{b}$, or $y = -\dfrac{a}{b}x + \dfrac{g}{b}$

$= -\dfrac{mg}{ng} = -\dfrac{m}{n}$



so we get that

if (s, t) is a integer solution to

$as + bt = g$ then so is

$$\left(s + kn, t - km\right) = \left(s + k\frac{b}{g}, t - k\frac{a}{g}\right)$$

Now suppose s', t' is some integer solution – i.e

$$as' + bt' = g$$

know

$$\hookrightarrow as + bt = g$$