So... the Euclidean algorithm will give you one solution to
\[ a \cdot s + b \cdot t = \gcd(a, b) \]
But there are others.
How do we get 'em all?
Let \( g = \gcd(a, b) \)

We want all integer points on the line

\[ a \times + b y = g \]

(We know \( s, t \) from the Euclidean Algorithm is on the line)
eg. \( \gcd(288, 51) = 3 \)

we want integer solutions to

\[288x + 51y = 3\]

we know \((x, y) = (-3, 17)\)

is a solution from the Euclidean algorithm.
The line $ax + by = g$ is $y = \frac{g-ax}{b}$ or $y = -\frac{a}{b}x + \frac{g}{b}$.
So we get that integer
if \((s, t)\) is a solution to
\[as + bt = g\]
then so is
\[(s + kn, t - km) = (s + k \frac{b}{g}, t - k \frac{a}{g})\]
Now suppose $s', t'$ is some integer solution, i.e.,

\[ as' + bt' = g \]

We want

\[ as + bt = g \]