Now prove that both $(c-b)$ and $(c+b)$ are squares.

$$c^2 - b^2 = a^2$$

$$(c-b)(c+b) = a \cdot a$$

SCRAP

$$\_\_\_ \cdot \_\_\_ = \underline{a \cdot a}$$

$$1 \cdot a^2 = a^2$$

$$4 \cdot 25 = 10 \cdot 10$$

$$2 \cdot 2 \cdot 5 \cdot 5 = 2 \cdot 5 \cdot 2 \cdot 5$$

$$2 \cdot 8 = 4 \cdot 4$$

common factors— oops.

$$9 \cdot 16 = 12 \cdot 12$$

$$3 \cdot 3 \cdot 4 \cdot 4 = 3 \cdot 4 \quad 3 \cdot 4$$

$$9 \cdot 4 = 6 \cdot 6$$

$$3 \cdot 3 \quad 2 \cdot 2 = 3 \cdot 2 \quad 3 \cdot 2$$

$$\frac{5^4\, 11^2}{S_1\, S_2} \,\Big|\, \frac{7^4}{T_1} = \boxed{5^2 \cdot 11 \cdot 7^2 \cdot 5^2 \cdot 11 \cdot 7^2}$$

A little excursion ....

Fundamental Theorem of arithmetic

any integer $n > 1$ can be written <u>uniquely</u> as

$$n = \prod_i p_i^{\alpha_i}$$

where $p_1, p_2, \ldots$ are the primes in increasing order.

eg $150 = 2^1 \cdot 3^1 \cdot 5^2 \cdot 7^0$

$\alpha_1 = 1, \ \alpha_2 = 1, \ \alpha_3 = 2, \ \alpha_n = 0$ for $n > 4$

$p_1 = 2 \qquad p_4 = 7$

$p_2 = 3 \qquad p_5 = 11$

$p_3 = 5 \qquad p_6 = 13$

$\vdots$

Go back to

$(c-b)(c+b) = a^2$

$\vdots$

$a = P_1^{\alpha_1} P_2^{\alpha_2} \cdots P_n^{\alpha_n}$

$(c-b)(c+b) = P_1^{\alpha_1} P_2^{\alpha_2} \cdots P_n^{\alpha_n} \cdot P_1^{\alpha_1} P_2^{\alpha_2} \cdots P_n^{\alpha_n}$

if $P_i \mid c-b$, then $P_i \nmid c+b$

$\therefore P_i^{\alpha_i+\alpha_i} \mid c-b$

so we can group the $\{P_1 \ldots P_n\}$

into $\{S_1 \ldots S_r\}$ and $\{T_1 \ldots T_m\}$

so that $\{S_1 \ldots S_r\} \cup \{T_1 \ldots T_m\} =$
$\{P_1 \ldots P_n\}$

and $\{S_1 \ldots S_r\} \cap \{T_1 \ldots T_m\} = \emptyset$

Then $c-b = S_1^{2\alpha_{S_1}} S_2^{2\alpha_{S_2}} \cdots S_r^{2\alpha_{S_r}} = \left(S_1^{\alpha_{S_1}} \cdots S_r^{\alpha_{S_r}}\right)^2$

$c+b = T_1^{2\alpha_{T_1}} T_2^{2\alpha_{T_2}} \cdots T_m^{2\alpha_{T_m}} =$

$\left[T_1^{\alpha_{T_1}} \cdots T_m^{\alpha_{T_m}}\right]^2$

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Thus if

$$(c-b)(c+b) = a^2$$

with $c-b$ and $c+b$ having no common factors, then $c-b$ and $c+b$ are squares

eg $a = n \cdot m$

and $c-b = n^2$, $c+b = m^2$, so

$$(c-b)(c+b) = n^2 \cdot m^2 = n \cdot m \cdot n \cdot m = a^2$$

so say $c+b = s^2$, $c-b = t^2$

with $s > t \geq 1$, s, t odd

with no common factors

$$(c+b) = s^2 \qquad \Longrightarrow \qquad 2c = s^2 + t^2$$

$$c - b = t^2$$

$$\boxed{c = \frac{s^2+t^2}{2}}$$

$$\frac{s^2+t^2}{2} + b = s^2 \Longrightarrow \boxed{b = \frac{s^2-t^2}{2}}$$

and

$$a = \sqrt{(c+b)(c-b)}$$

$$= \sqrt{s^2 \cdot t^2} = st$$

$$\boxed{a = st}$$

Bottom Line

All P.P.T.

have the form $\left(st, \frac{s^2-t^2}{2}, \frac{s^2+t^2}{2}\right)$

with $s > t \geq 1$

s, t odd with

no common

factors.