Mathematical induction:

you want to prove some formula $F(n)$ that depends on a number $n$.

Proof:

step 1: base case. Prove your formula works for some small $n$ (eg 0, 1, 2)
Step 2: Assume your formula works for $n-1$,

\[ F(n-1) = \text{whatever is true}. \]

Step 3: Use that $F(n-1) = \text{whatever plus other stuff you (hopefully!) know}$ to show that $F(n) = \text{the formula you want.}$
\[ T_n = T_{n-1} + n \]

\[ T_m = \frac{m(m+1)}{2} \]

\[ T_2 = \frac{2(2+1)}{2} = \frac{6}{2} = 3 \]

\[ T_{n-1} = \frac{(n-1)(n-1+1)}{2} = \frac{n^2-n}{2} \]
\[ T_n = T_{n-1} + n \]

\[ \frac{n(n+1)}{2} = \frac{n^2 - n}{2} + n \]

\[ \frac{n^2 + n}{2} = \frac{n^2 - n}{2} + \frac{2n}{2} \]

\[ \frac{n^2 + n}{2} = \frac{n^2 - n + 2n}{2} \]

\[ \frac{n^2 + n}{2} = \frac{n^2 + n}{2} \]

\[ T_n = \frac{n(n+1)}{2} \]
\[ T_{aq} = \frac{q_1(91H)}{2} \]

\[ T_{aq} = T_{aq0} + 91 \]
\[ T_{10} = \frac{10(10+1)}{2} \]

\[ T_{10} = 55 \]
Chpt 2.

Everyone (individual problem): 2.2

group #
A 2.1
B 2.7
C 2.6
D 2.5
Pythagorean triples

No common factors

(3, 4, 5)
(5, 12, 13)
(8, 15, 17)
(7, 24, 25)

(4, 8, 10)
(9, 12, 15)
(12, 16, 20)
(15, 20, 25)
(18, 24, 30)
We want primitive Pythagorean triples (PPTs) i.e., $a, b, c$ so that $a^2 + b^2 = c^2$, but $a, b, c$ have no common factors.
Note that $c$ is always odd, and one of $a, b$ is even and one is odd.

If both $a, b$ were even, then $c$ would be even, and $(a, b, c)$ would have a common factor of 2, so not a PPT.
If $a, b$ were both odd, the $c$ would be be even.

ie.

$$a = 2m + 1, \quad b = 2n + 1$$

for some $m, n, k$

$$c = 2k$$

Then

$$(2m+1)^2 + (2n+1)^2 = (2k)^2$$

$$4m^2 + 4m + 1 + 4n^2 + 4n + 1 = 4k^2$$

$$2m^2 + 2m + 2n^2 + 2n + 1 = 2k^2$$

Even

Even

Odd

Can't have odd = even.

therefore

must have one $a, b$ even and one odd.