5.4 part c)

$k = \gcd(m, n)$

$m = kx$ $\quad \gcd(x, y) = 1$

$n = ky$

$$\text{lcm} = \frac{m \cdot n}{\gcd} = \frac{kx \cdot ky}{k} = kxy = L$$

Let P = some other comm. mult.

$P = am = ln$

$P = a\overbrace{kx}^{=m} = b\overbrace{ky}^{=n}$

$ax = by$

$x / by \Rightarrow x | b$

so $b = cx$

Thus, $P = cxky$ so L / P

and $L = xky$ giving $L \leq P$

$\therefore L$ is <u>least</u> comm. mult.

$m:30;\ n:45$

$m = 15 \cdot 2$

$n = 15 \cdot 3$

$K = 15$

$x = 2$

$y = 3$

$\gcd = K = 15$

$LCM = Kxy = 90 = L$

$c:2$

$P = 180 = 30 \cdot 6 = 45 \cdot 4$

$cxKy$

$(15 \cdot 2 \cdot \underset{(3 \cdot 2)}{6}) = (15 \cdot 3 \cdot \underset{(2 \cdot 2)}{4})$

use: $\gcd(m,N) = \prod p_i^{\min(\alpha_i,\beta_i)}$

$\operatorname{lcm}(m,N) = \prod p_i^{\max(\alpha_i,\beta_i)}$

$M = \prod p_i^{\alpha_i}$

$N = \prod p_i^{\beta_i}$

so... $\prod p_i^{\max(\alpha_i,\beta_i)} = \dfrac{\left(\prod p_i^{\alpha_i}\right)\left(\prod p_i^{\beta_i}\right)}{\prod p_i^{\min(\alpha_i,\beta_i)}}$

$\rightarrow \left[\prod p_i^{\max(\alpha_i,\beta_i)}\right]\left[\prod p_i^{\min(\alpha_i,\beta_i)}\right] =$

$\left(\prod p_i^{\alpha_i}\right)\left(\prod p_i^{\beta_i}\right)$ mult exp = addition

looking at $\max(\alpha_i,\beta_i) + \min(\alpha_i,\beta_i) = (\alpha_i+\beta_i)$

looking at exponents of p_i's ✓