ROUND ROBIN TOURNAMENTS WITH ONE BYE

DALIBOR FRONCEK

University of Minnesota Duluth

Many sport competitions are played as 2-leg round robin tournaments with $2n$ or $2n-1$ teams. These tournaments are typically scheduled in such a way that a schedule for a 1-leg tournament with $2n-1$ rounds is repeated twice with the home team being switched in the second leg. By a *round* we mean a collection of games in which each team plays at most one game. A team that does not play a game in a particular round is said to have a *bye* in that round.

The home and away games of every team should interchange as regularly as possible provided that each team meets every opponent once at its own field and once at the opponent's field. The best *home-away pattern* (HAP) is indeed one with no two consecutive home or away games (such a pair of games is called a *break* in the schedule). Obviously, we can never find a schedule for $2n$ teams with $2n-1$ rounds with no breaks—in this case the teams that start the season with a home game would never meet. Therefore, looking at HAPs, the best schedule is one with the minimum number of breaks. This number in a 1-leg round robin tournaments is $2n-2$, as proved by de Werra.

We will show that if each team has exactly one bye, then we can construct schedules with no breaks, and that these schedules are unique. The constructions are surpisingly simple and involve just basic notions from graph theory and modular arithmetic. No previous knowledge of graph theory is needed as the notions are elementary and will be explained.

This talk is based on a joint work with Mariusz Meszka, University of Science and Technology, Krakow, Poland.