

EIGHT QUEENS AND MORE

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In how many ways can q queens be placed on an $n \times n$ chessboard so no two queens attack each other? What about other chess pieces, like bishops or nightriders (a fairy chess piece)? These questions generalize the famous Eight Queens Problem, which asks for the number of ways to place 8 nonattacking queens on an 8×8 chessboard, to any number of identical pieces on any sized square board.

Seth Chaiken, Chris Hanusa, and I have treated the questions by means of a hyperplane-arrangement generalization of Ehrhart's theory of counting lattice points in a convex polytope. The number $N_q(n)$ of nonattacking configurations is a function of n . By counting $1/k$ -fractional points in a convex polytope we showed that $N_q(n)$ is given by a cyclically repeating series of p polynomials of degree $2q$, provided that the piece's moves are unlimited, like those of a queen, rook, or bishop but not a king or knight.

An upper bound for the period p is the least common denominator of the "vertices" of the combined polytope and hyperplanes; this number depends on the Kronecker product of two matrices and can sometimes be computed as a rather complicated least common multiple. This helps us to estimate the value of p .