HOW HIGH? HOW FAST? HOW LONG? MODELING WATER ROCKET FLIGHT WITH CALCULUS

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ABSTRACT: We describe an easy and fun project using water rockets to demonstrate applications of single variable calculus concepts. We provide procedures and a supplies list for launching and videotaping a water rocket flight to provide the experimental data. Because of factors such as fuel expulsion and wind effects, the water rocket does not follow the parabolic model of a textbook projectile, so instead we develop a one-variable height vs. time polynomial model by interpolating observed data points. We then analyze this model using methods suitable to a first semester calculus course. We include a list of questions and partial solutions for our project in which students use calculus techniques to find quantities not apparent from direct observation. We also include a list of websites and other resources to complement and extend this project.

KEYWORDS: Water rockets, projectiles, single variable calculus, modeling, curve fitting, three dimensional trigonometry, practical applications, classroom project, hands on activity, active learning, graphing calculator, CAS.

1 OVERVIEW AND OBJECTIVES

Water rockets provide an easily implemented, engaging activity that allows students to experience first-hand how scientists use mathematical modeling and the tools of calculus to determine properties not apparent from the raw data alone. In this activity, students develop a model for the rocket height of a single water rocket launch after videotaping it in front of a building of known dimensions and then use calculus concepts to analyze the rocket performance. Water rockets (see Figure 1) are cheap, re-usable, easy to launch, and have a very high fun-to-nuisance ratio. They also provide an example of some of the fundamental aspects of projectile behavior, while factors such as the propulsion mechanism and wind effects give them more complex flight paths than those of simple projectiles. Although there is a lot of available information about combustible-fuel model rockets (see Section V), they move too fast for easy measurements and require too much space. Water rockets are not as expensive nor logistically difficult as combustible-fuel model rockets, yet they have just enough complexity in their flight paths to provide an opportunity to put the skills and concepts learned in calculus to work in a substantial way.

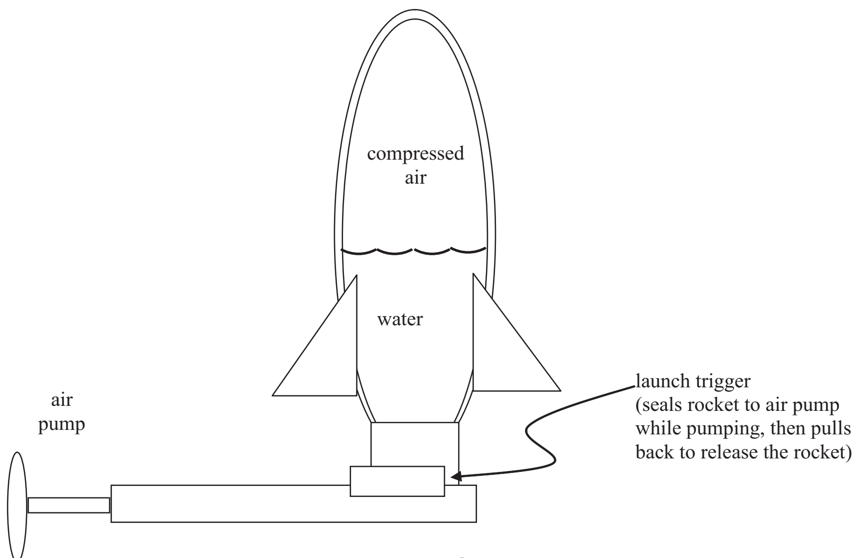

Figure 1. Water rocket components.

One of the main goals of the activity is to provide a physical context incorporating many central calculus concepts. Another objective is for students to create and interpret a model and realize that it provides information not available from the experimental data alone. In particular, once the rocket rises above the backdrop of the building, there is no longer a frame of reference to estimate its height experimentally, thus necessitating a mathematical model. Also, questions such as finding the impact velocity and maximum height can only be addressed with a model; these quantities cannot be determined from the raw data or even from the videotape. Students also explore some of the limitations of such modeling efforts.

Most standard calculus (or physics) textbooks include a parabolic model describing the flight of a projectile. In the one-dimensional case, the function $s(t) = v_0 t - \frac{1}{2}gt^2$ models the vertical position of a projectile with respect to time t, where $g = 32.2$ ft/sec^2 is Earth's gravitational constant and v_0 is the initial velocity. Other than the initial acceleration, this model considers only gravity acting on the rocket, and ignores air resistance, for example.

Experiments demonstrating this parabolic model often require access to a large windless space (such as a hangar) and a mechanism to measure the initial velocity, neither of which was readily available to us. Rather than trying to design an experiment to demonstrate the parabolic model, or to fit a parabolic model to the experimental data, we decided to explore the vagaries introduced by wind (irregular gusts, not just air resistance) and varying propulsion by modeling individual water rocket launches.

To collect data, students first launch the rockets against a backdrop of known height, such as a building, and videotape the experiment. Then, they use the dimensions of the backdrop, the position of the camcorder, and an analysis of the videotape to determine the rocket height at several times. With these data points and a graphing calculator or CAS, students can use curve fitting tools to construct a higher order polynomial model of the height of the rocket as a function of time. We use polynomial interpolation since it is easier to motivate within our curriculum than, for example, a least squares fit, and a simple interpolation command is available in Maple. However, any curve fitting technique available on a graphing calculator or CAS would work as well.

This project is intended for students taking single variable differential calculus. Students must be able to determine and interpret first and second derivatives. Substantial trigonometry and three-dimensional geometry are required for estimating the heights from the videotape of the rocket against the building, which provides an excellent review and valuable context for these mathematical foundations. Students will need to solve several equations in several variables for the polynomial interpolation, or use a graphing calculator or CAS with polynomial interpolation capability. Advanced students familiar with calculus in three dimensions may do more advanced modeling and analysis, such as developing a parameterized curve for the rocket's flight path and using it to consider such aspects as arc length and tangential velocities (see [1]).

We typically use this activity as a midterm project, with the work extending into the second half of the course and accounting for roughly 15%

of the final grade. Students work in small groups, with about two to four students per group. We spread the project over several weeks, with 'check point' deadlines for various steps of the project. These steps break down as follows: getting a good video recording of a usable launch, finding the actual heights and fitting a curve, and then completing the final analysis. We don't devote much in-class time to the project, the idea being that this application simulates, in a small way, how students will independently address career-related applications requiring a synthesis of what they have learned throughout their studies. Our students do request some assistance outside of class, but usually for brief, technical questions. We use Maple in our courses, and the final form of the project is a well-documented Maple worksheet with expository portions entered as text. Assessing student computational work is streamlined by the use of Maple. It is clear from the graphs whether the students have found well fitting curves, and then we simply enter those curves into our own Maple code to generate the solutions for many components of the project. It requires somewhat more time to evaluate students' expository interpretations of their results.

We have found that our students have fun with this project, and in the process, gain a valuable experience in applying mathematics. They enjoy the opportunity to play outside with the rockets and, with this, the activity builds a camaraderie among classmates that carries over into more structured studies. The analysis requires a valuable synthesis of mathematics ranging from high school trigonometry through the current calculus course, and thus reaffirms for students the importance of their mathematical education. It confirms for students the value of a CAS for computationally intensive tasks associated with real data. The project also gives students the chance to surmount some of the challenges that inevitably arise when collecting and analyzing real data as opposed to simply stepping through a carefully constructed textbook problem, and thus better prepares them for their careers. For all of these reasons, our students respond positively to this project.

The following sections contain the details of how we have carried out this activity in our classes. Section II provides a supplies list and a description of launch logistics, and then Section III contains a handout to guide student work, followed by a sample launch with solutions in Section IV. If actually having students perform the experiment is unfeasible, this activity may be done by giving the students the data provided in Figure 4 and Table 1 in Section IV, although students are much more engaged by launching the rockets themselves and then collecting and analyzing their own data. We provide some additional resources in Section V.

2 SUPPLIES AND PROCEDURE

Supplies needed

- Water rockets – obtain several extra in case of launch damage or defective rockets.

  Water rockets are seasonally available in many toy stores or may be ordered on-line (well in advance, since the sources are not always reliable). We have used various models over the years, and all have worked more or less reasonably well. We usually pay less than $5.00 each for the rockets.

- Tape measure and (optional) 100 foot length of rope.

  Although metric units are typical for scientific work, most blueprints in the United States use standard units, as do most readily available long tape measures, so we have used standard units for this activity. Since the distances may be longer than the tape measure, a pre-measured length of rope can be handy.

- Blueprints, if available, for a nearby three-story building – at least the basic dimensions of the building must be known.

  We found blueprints of the building façade in the physical plant office of our institution. Since we had to reduce the scale when we copied the blueprint, we marked an inch line on the blueprint, with the scale next to the inch line, so that the diminished inch line became the new scale on the reduced photocopies we gave our students.

- Camcorder with videotape, or digital camcorder and disks.

  Camcorders were available through the media center of our library, and videotapes or disks may cost $4-$10 depending on the type. We arranged for students to check out equipment from the library, and they bought their own tapes/disks.

- Editing capability to view the video frame by frame, if available.

  We needed to allocate additional time to be trained on this software the first time we used it, and we had to understand it well enough to give simple instructions to our students. Again, our media staff were most helpful, and made themselves available to our students if any technical issues arose during the project.

- Stopwatch, if unable to view the video frame by frame, and at least a VCR with a pause button.

We used this method in the years before we had digital video, and while it resulted in fairly sloppy data, the project still seemed to work fine. Students appeared to recognize that the modeling process was fundamentally sound and would be the same (as in fact it is) even with more sophisticated means of data collection. That said, if frame by frame viewing is available, it is worth the effort to use it.

Launching procedure

1. Choose an appropriate backdrop, such as a building with known height. This activity is more interesting if the building is about three stories tall. The rocket will then appear to rise above the building so that its maximum height has to be estimated using calculus.

2. Before launching the rockets, measure distances from the camcorder site to the launch site and from the launch site to the building. The camcorder and launch site should be on a line perpendicular to the building through an easily identifiable location on the building. We stood about 80 feet from the building. Our building is on a slight rise above the flat lawn where we stood with the camcorder, so we measured the stairs leading up to the building to estimate the vertical offset. A flat area is easiest to work with and should be used if at all possible. Otherwise, some means of estimating the vertical offsets of the launch and landing sites is necessary. A simple sighting mechanism (e.g. tripod with a level) and a marked tall pole (e.g. length of PVC pipe with electrical tape at one foot increments) might suffice, although we have never tried it. Be aware that this means using a measurement method for the launch and landing data points that is different from that used for the mid-flight data points, and this can create anomalies in the model. In general, measurement error can be significant in this experiment—if possible, take each measurement twice and average the results.

3. Launch the rocket and record the flight on the camcorder. A successful launch is one in which the rocket appears in front of the building at both the beginning and the end of the flight and appears to rise above the roofline at its maximum height. Several launches may be necessary to achieve this. Having someone say, for example, "This is the third trial" as the camcorder operator begins to record the launch will help identify the different trials when viewing the tape later.

4. Once the rocket has landed, measure the distances from the camcorder

to the landing site, and from the launch site to the landing site. Again, measure twice and average the values.

5. View the videotape of the successful launches, decide which one to use for the model, and then gather at least six data points during the flight of the rocket. The blueprints of the building in the background will help to determine the vertical and horizontal distances that the rocket has traveled in reference to the building. If blueprints are unavailable, then make estimates based on the known height of the building, and take measurements for the horizontal distances. Most camcorders record at about thirty frames per second, which determines the timing. (This information can usually be found in the instruction manual.) If unable to view the tape frame-by-frame, then use a stopwatch and the pause button to estimate as accurately as possible.

3 MODELING AND ANALYSIS

Below is the handout we give our students when we use this project in our classes. We typically do this activity as a group project, dividing a class of roughly 24 students into groups of 2 to 4 students each. We provide each group with a handout, rocket, and blueprints. Each group is responsible for their own measurement tools and video tape or disk. We discuss the launching procedure with the groups, and indicate where to check out equipment and receive technical support, and then let them work quite independently on the project.

Rocket Flight Analysis Handout

Group Members:____________________________

Section: __________________________________

Using the information extracted from the videotape, develop a polynomial to model the rocket flight. Keep in mind that the videotape stills do NOT give the actual rocket heights above ground level. They only show how high up the building the rocket appeared to be from where the camcorder operator was standing. *Determining the actual heights is a significant part of this activity.* You may find it helpful to review similar triangles and the laws of sines and cosines.

1. Draw a ground diagram of your launch site. Label the building, camcorder location, launch site and landing site. Include measurements of all distances on the diagram. Be sure to note the height of the camcorder.

2. View the videotape, select several stills at times when the rocket was ascending and when it was descending, and note the times at which they were taken. Compare the stills to the building blueprint and locate the rocket on the blueprint at each time. Measure the blueprint (be careful of scale!) to estimate the horizontal distances from the fixed point on the building and vertical distances above ground level where the rocket appears in the video still.

3. Provide a table giving the times and apparent rocket positions against the building:

Apparent Position Table		
Time	Horizontal Distance on Building	Vertical Distance on Building

4. Create a 'side view' diagram as in Figure 2, where Q is the height of the camcorder. This shows the similar triangles used to determine the actual heights s_i of the rocket from the perceived heights q_i of the rocket against the building at time t_i .

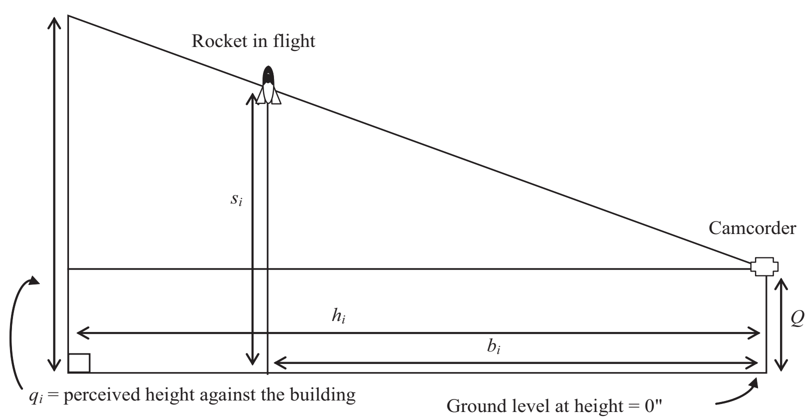

Figure 2. Side view showing similar triangles for computing heights.

5. Make a second, more detailed, ground diagram as in Figure 3. Use your measurements to fill in the values for D_1, D_2, L, C, E, and use trigonometry to determine the values of the angles A and B. The variables h_i and b_i, which you will determine in the next question, are the same as those in Figure 2.

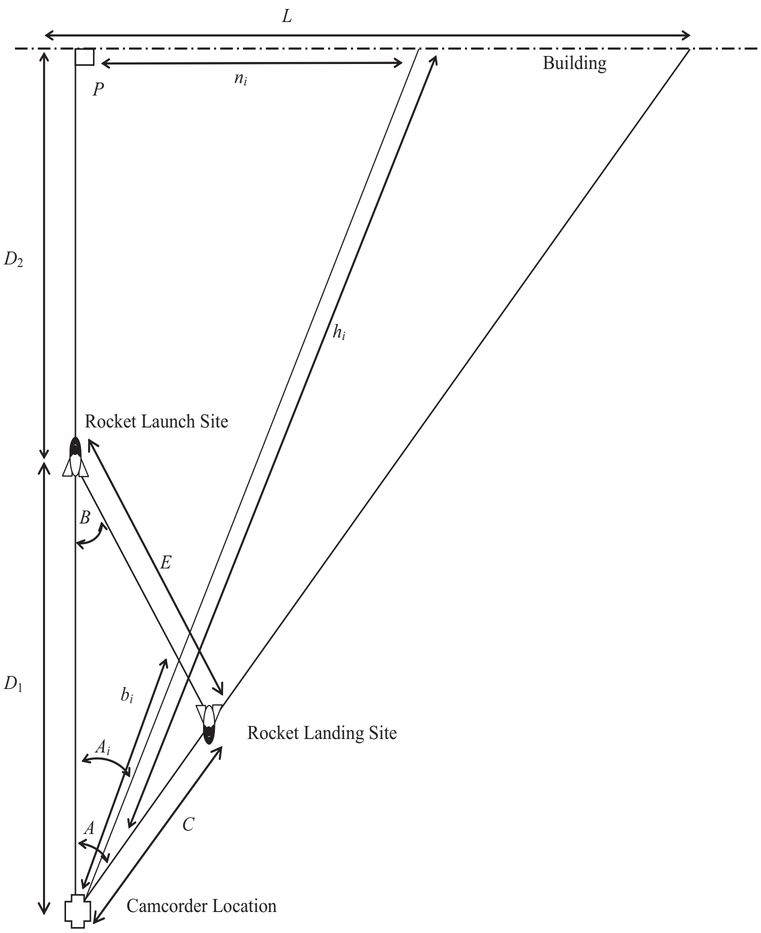

Figure 3. Detailed Ground Location Diagram for Finding Actual Heights.

6. Use similar triangles and trigonometry to determine the values of A_i, b_i, h_i, n_i, and finally s_i, the actual height of the rocket, for each time t_i. Use the table below to record these values at each of the times you collected a data point.

Actual Positions							
i	t_i	n_i	q_i	A_i	h_i	b_i	s_i

Carefully show how you computed these values. You may assume that the rocket flight path remains in the vertical plane through the line between the launch and landing sites. Note that n_i and q_i are just the perceived horizontal and vertical distances on the building from Question 3.

7. Find a polynomial to model the height of this rocket at time t by interpolating data points. You will probably need to experiment with both the degree of the polynomial and the choice of data points to use before you find a satisfactory model. Recall that you need $n + 1$ data points to determine an n^{th} degree polynomial. You may find that a polynomial of low degree fails to capture wind effects, while a polynomial of high degree may flatten out too much at the top or introduce strange wiggles. Compare several possible models and explain your final choice of polynomial and data points. Provide a plot of height versus time with the actual height data points superimposed over a graph of your modeling function.

8. Approximate the initial and impact velocity and acceleration using your model. Describe the calculus concepts needed to find these. Are these quantities greater at takeoff or at landing? Why might this be? Do you think your model accurately reflects reality? How do these accelerations compare to the acceleration due to gravity? How do they compare to the acceleration of a space shuttle at takeoff (29 m/sec^2), a cheetah at takeoff (7.8 m/sec^2), or a parachute at landing (35 m/sec^2) (see [7])? Note units!

9. Use your model to estimate the maximum height attained by your rocket and the time it was attained. Describe the calculus concepts that you applied to answer this. What are the velocity and acceleration at the maximum height? Why?

10. Also, use your model to find estimates of the time(s) when the rocket is 100 inches above the ground, and then use the model to estimate the velocity and acceleration of the rocket then.

11. Comment on your model. How well does it fit the data points you have? Do you think it provides an accurate description of the rocket flight? Why or why not?

12. What would you do to develop a more accurate model of this particular rocket flight? What would you do to develop a generic model (one that would work for any launch) for this type of water rocket?

13. What are some causes of variation from launch to launch? What variables could you control for and how could you do it?

14. (*Extra Challenge*) Most calculus texts contain a simple parabolic model for the height of a projectile. After looking this up, describe the information you would need in order to use this model. Can you get that information from your data? Or at least a reasonable estimate of it? Use your best estimates to model your rocket flight with this model. Provide a plot of height versus time with the actual height data points superimposed over a graph of this modeling function. Do you think this is a better or worse model than the one you developed? Why? Is there anything that one model accounts for that the other does not?

4 PARTIAL SOLUTIONS

The solutions below are based on the setup and raw data from one of our rocket launches. All table values are given to three significant digits. Data from other launches will result in different models.

Question 1. Ground diagram (see Figure 4). The rocket was launched from ground level. The camcorder was at a height of 60″ above ground level.

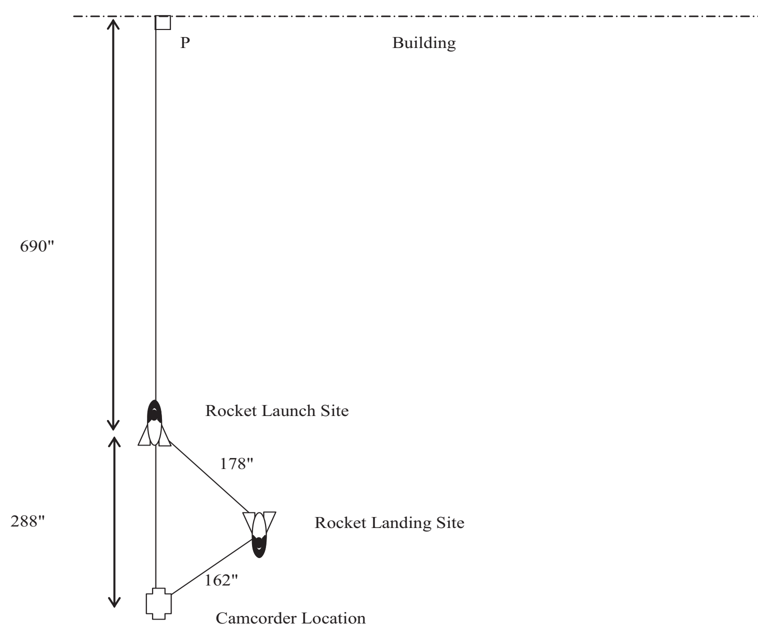

Figure 4. Ground Diagram for the Rocket Launch. Measurements are in inches. Drawing is *not to scale*.

Question 3. Table 1 gives the locations of the rocket against the building blueprint from the vantage point of the camcorder operator. The heights are up from ground level, and the horizontal distances are to the right of the point marked "P" in Figures 3 and 4.

Time	**Horizontal Distance on Building**	**Vertical Distance on Building**
0 sec	0″	0″
0.233 sec	24.0″	216″
0.367 sec	48.0″	372″
0.500 sec	72.0″	504″
1.67 sec	336″	516″
1.83 sec	360″	396″
1.97 sec	384″	288″
2.10 sec	408″	180″
2.37 sec	656″	0″

Table 1. Apparent positions against building.

Question 4. Our value for Q is 60″ in Figure 2.

Question 5. Our values for D_1, D_2, L, C, and E in Figure 3 are 288″, 690″, 656″, 162″, and 178″, respectively. Then, we use the law of cosines to find A and B:

$A = \arccos\left(\frac{162^2+288^2-178^2}{2(162)(288)}\right) = 0.591$ radians,

$B = \arccos\left(\frac{178^2+288^2-162^2}{2(178)(288)}\right) = 0.532$ radians.

Question 6. The horizontal distances n_i for i from 1 to 7 come from the observed rocket positions. Note from Figure 3 that $n_0 = 0$, and since the line from the camcorder through the launch site forms a right angle with the building, then $n_8 = 978 \tan(A)$.

The angles A_i can now be found by $A_i = \arctan(n_i/978)$, and then $h_i = 978/\cos(A_i)$. Since $b_0 = 288$, proportions from the law of sines give that $b_i = 288 \sin(B)/\sin(\pi - (B + A_i))$.

The actual rocket heights at launch and landing time are $s_0 = 0$ and $s_8 = 0$, respectively. The inflight heights can be computed using similar triangles as shown in Figure 2, so $s_i = ((q_i - 60)\, b_i/h_i) + 60$. Table 2 gives all of these values. Times are in seconds, the A_i's are in radians, and all other measurements are in inches.

i	t_i	n_i	q_i	A_i	h_i	b_i	s_i
0	0	0	0	0	978	288	0
1	0.233	24.0	216	0.0245	978	277	104
2	0.367	48.0	372	0.0490	979	266	145
3	0.500	72.0	504	0.0735	981	257	176
4	1.67	336	516	0.331	1030	192	145
5	1.83	360	396	0.353	1040	189	121
6	1.97	384	288	0.374	1050	186	100
7	2.10	408	180	0.395	1060	183	80.7
8	2.37	656	0	0.591	1180	162	0

Table 2. Actual rocket positions.

Question 7. The data points corresponding to times t_0, t_2, t_3, t_6, and t_8 (we chose these points after experimentation with various sets of points) give $P(t) = -14.5t^4 + 112t^3 - 408t^2 + 530t$ as a quartic polynomial model of the heights of the rocket at time t. This polynomial can be found by solving a system of linear equations or by using the polynomial interpolation functions of a graphing calculator or CAS. Note that all of the nine data points can be interpolated using a degree 8 polynomial,

$$Q(t) = -110t^8 + 998t^7 - 3670t^6 + 6970t^5 - 7220t^4 + 4090t^3 - 1500t^2 + 646t.$$

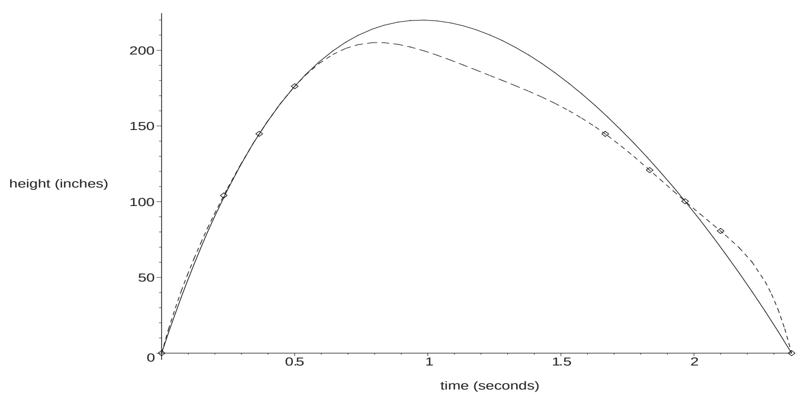

Figure 5. Quartic Model (solid) and Degree 8 Model (dashed) and Data Points.

Although $Q(t)$ does a better job "fitting" all of the data points (see Figure 5), it may not as useful a model as $P(t)$. In particular, the rocket rose significantly higher than the third in-flight data point, but $Q(t)$ does not reflect that as well as $P(t)$ does.

Question 8. Here, $P'(t) = -58.0t^3 + 336t^2 - 815t + 530$ and $P''(t) = -174t^2 + 671t - 815$. Thus, the initial and impact velocities estimates are given by $P'(0) = 530$ in/sec (30.1 mph) and $P'(2.37) = -288$ in/sec (-16.4 mph), and the initial and impact accelerations estimates are $P''(0) = -815$ in/sec^2 (-67.9 ft/sec^2) and $P''(2.37) = -201$ in/sec^2(-16.8 ft/sec^2). This model reflects that the launching velocity and acceleration were higher than at landing. This may be due to the effect of the wind loft on the empty (hence light) rocket at the end of the flight.

Question 9. The zero of $P'(t)$ is 0.977 seconds, and $P(0.977)$ gives an estimated maximum rocket height of 220 inches or 18.3 feet. The velocity is 0.000 in/sec and the acceleration is -325 in/sec^2 at this time. The fact that a rocket that actually only rose 18.3 feet appeared to rise above the building from where it was observed is a good lesson in perspective!

Question 10. The solutions to $P(t) = 100$ are 0.225 sec. and 1.97 sec. Then, the velocity and acceleration when the rocket is rising are estimated by $P'(0.225) = 363$ in/sec and $P''(0.225) = -673$ in/sec^2, and the velocity and acceleration when the rocket is falling are estimated by $P'(1.97) = -216$ in/sec and $P''(1.97) = -168$ in/sec^2.

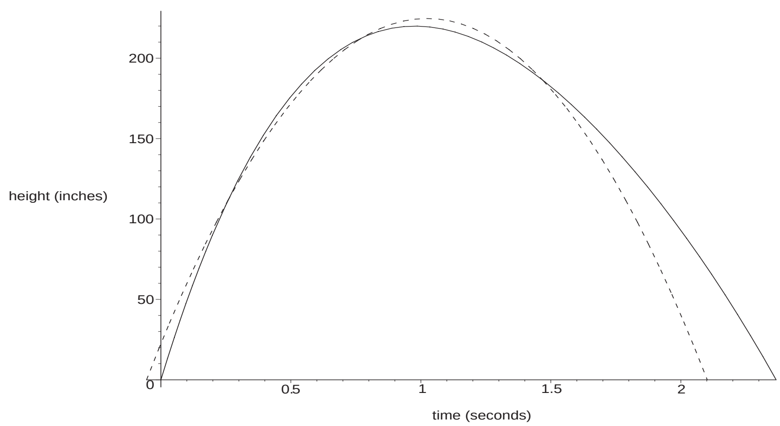

Figure 6. Quartic Model (solid) and Standard Parabolic Model (dashed).

Question 14. (*Extra Challenge*) We make a comparison with the standard parabolic model for the height of a projectile, $s(t) = vt - \frac{1}{2}gt^2$, to get some sense of what effect the wind had on the height. We used the second and third data points to provide a velocity estimate of 305 in/sec at time t_1, and shifted the standard model, since this seemed to give the most useful parabolic model, $s(t) = 305(t - 0.233) - 193(t - 0.233)^2 + 104$. See Figure 6.

We estimated the velocity at t_1 instead of t_0 because there is a (short) thrusting phase while the water is expelled, so sometimes later data will give a better sense of the velocity. However, since we used t_1 instead of t_0, we needed to shift the parabolic model horizontally and vertically. In general, the formula for the shifted model is $s(t) = v(t-t_i) - \frac{1}{2}g(t-t_i)^2 + s_i$, where v is the velocity estimate and s_i is the height at time t_i. From Figure 6, we can see that the rocket actually remained airborne longer than predicted by the parabolic model, thus demonstrating the effect of additional lift of the wind during the launch.

RESOURCES

1. Ashline, George, Joanna Ellis-Monaghan, and Alain Brizard. 2003. Water Rockets in Flight: Calculus in Action. *UMAP/ILAP Modules 2002-2003: Tools for Teaching*. 151-188.

This longer paper extends the water rocket experiment to demonstrate some of the concepts of multivariable calculus. Polynomial interpolation is used to calculate the X, Y, and Z coordinate functions of the rocket as a function of time during its entire flight, and methods from multivariable calculus offer flight analysis and estimation of the rocket's maximum height and flight path curvature not apparent from direct observation. Examination of first and second time derivatives of the rocket coordinates allows us to identify the thrusting, coasting, and recovery stages of the rocket flight, and comparison to the parabolic model shows the effects of the wind.

2. Benson, Tom. 2004. Homepage of NASA Glenn Research Center: Beginner's Guide to Aeronautics. `http://www.grc.nasa.gov/WWW/K-12/airplane/index.html` [2005, August 20].

This website offers a "Beginner's Guide to Aerodynamics" and a "Beginner's Guide to Model Rockets." The model rocket information includes some of the basic mathematics and physics concepts that affect model rocket design and flight. The site is intended to offer K-12 teachers, students, and others with basic background information on aerodynamics and propulsion.

3. Estes Industries. 2005. Homepage of Estes Industries Publications. `http://www.esteseducator.com/cfusion/publications.cfm` [2005, August 20].c

Estes Industries for over 40 years has been "...the world's leader in the manufacturing of safe, reliable, and high quality rocket products" and currently offers a variety of "new and innovative educational products." This site is a comprehensive source of information about how model rocketry can be integrated into the classroom, and offers links to a number of different publications (in PDF format) on model rocketry, ranging from order forms to teachers guides to elementary mathematics information on rocket flights.

4. Flath, Daniel, Cliff Stoll, and Stan Wagon. 2005. Rocket Math. 35 (4): 262-273. *The College Mathematics Journal*.

This article addresses two questions concerning a model rocket experiment. The first considers ground observers determining the height of a rocket using only the angles of inclination between the ground and the rocket. The second involves measurement observational error, and ways to measure observed angles to estimate rocket height in a statistically sensible way.

5. Nave, C. Rod. 2001. Homepage of HyperPhysics. `http://hyperphysics.phy-astr.gsu.edu/hbase/hframe.html` [2005, August 20].

This website is "an exploration environment for concepts in physics which employs concept maps and other linking strategies to facilitate smooth navigation." It contains useful information on rocket propulsion.

6. Van Milligan, Tim. 2005. Homepage of Apogee Rockets. `http://www.apogeerockets.com/education/index.asp` [2005, August 20].

The goal of this site as a model rocket educational guide is to "... provide useful information about using model rockets in the classroom." This includes tips on teaching with rocketry, and background and links to other sites for more technical information on model rockets and real rockets.

7. Vawter, Richard. 2004. Homepage of Richard Vawter. `http://www.ac.wwu.edu/~vawter/PhysicsNet/Topics/Kinematics/Acceleration-Values.html` [2005, August 20].

This site offers some common acceleration values to compare to rocket estimations.

BIOGRAPHICAL SKETCHES

George Ashline received his BS from St. Lawrence University, his MS from the University of Notre Dame, and his PhD from the University of Notre Dame in 1994 in value distribution theory. He has taught at St. Michael's College since 1995. He is a participant in Project NExT, a program created for new or recent PhD's in the mathematical sciences who are interested in improving the teaching and learning of undergraduate mathematics. He is also actively involved in professional development programs in mathematics for elementary and middle school teachers.

Joanna Ellis-Monaghan received her BA from Bennington College, her MS from the University of Vermont, and her PhD from the University of North Carolina at Chapel Hill in 1995 in algebraic combinatorics. She has taught at St. Michaels College since 1992. She is a proponent of active learning and has developed materials, projects, and activities to augment a variety of courses. She participates in outreach programs sharing mathematics with community members ranging from elementary school children to senior citizens.